On a Liu-Wang Speech Recognition Model using an Orthogonal Polynomial Representation

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Abstract

Advanced speech information processing systems require further research on speaker-dependent information. Recently, a specific system of discrete orthogonal polynomials $\{\phi_r^L(l);\, l = 1, 2, ..., L\,\}_{r=0}^{R}$, has been encountered to play a dominant role in a segmental probability model recently proposed in the speaker-dependent feature extraction from speech waves and applied to text-independent speaker verification. Here, these *speech polynomials* are shown to be the shifted Chebyshev polynomials on a discrete variable $t_r(l-1, L)$, whose structural and spectral properties are discussed and reviewed in the light of the recent discoveries in the field of discrete orthogonal polynomials. Consequently various considerations and findings are shown which could greatly simplify the algorithms inherent to the speaker recognition methods and applications.

1 Introduction.

The classical orthogonal polynomials have been shown to play a relevant role in speech science; particularly, in research on extraction of speaker-dependent features from speech waves. This is the case of the Legendre polynomials which have been used for speech recognition, speech enhancement, speaker adaptation, ... (see e.g. [4, 6, 9, 8]). The discrete polynomials were firstly used in speech recognition performance by means of the orthogonal-polynomial-compression technique (Levitt & Rabiner, 1971 [14]; Levitt & Neuman 1991 [13]), which allows us to independently compress different aspects of the speech spectrum. Indeed, each polynomial corresponds to a different feature of the short-term speech spectrum; for example, the polynomials of first and second degrees correspond to the average slope and quadratic curvature of the spectrum.

Recently, the discrete polynomials have been used in the research of speaker recognition to face both speaker verification and speaker identification problems. Indeed, it has been proposed (Liu & Wang, 1996 [11]) a segmental probabilistic model which is based on an orthogonal polynomial representation of speech signals. Contrary to the conventional frame-based probabilistic models, the Liu-Wang model concatenates several consecutive frames with similar characteristics into an acoustic segment and represents it by an orthogonal polynomial function. Thus, the speech signal is composed of L successive $N-$dimensional feature vectors; that is, a set of N trajectories of length L. Each trajectory is represented by

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a polynomial function. Also, Liu and Wang propose an iterative, self-consistent procedure that performs recognition and segmentation processes for estimating the segment-based speaker model. Moreover, they illustrate its validity not only in the text-dependent speaker recognition, where the speaker is required to issue a predetermined utterance, but also in the text-independent recognition methods which do not rely on a specific text being spoken. These methods, whose aim is to verify the identity of a claimed speaker, have a training phase and a verification phase. For a given speech signal of a specific speaker, the number of segments, the length of each segment and the appropriate segmental probabilistic model are determined in the training phase. The performance of the model depends on the mixture number (i.e., number of acoustic segments used for modeling the speaker's voice characteristics) and the degrees of the orthogonal polynomials. This degree affects the accuracy of the model and controls the type of the basic segment for the model and its characteristics. In fact, the degree of the orthogonal polynomial used in the model determines the smallest length of the partitioned segment of a given speech signal. Moreover, the degree and the algebraic properties of the discrete orthogonal polynomials used are crucial for the efficiency (computation time and memory storage) and accuracy of the model. Furthermore, the discrete orthogonal polynomials that have naturally encountered in some speaker recognition methods (to be called hereto-forth *speech* polynomials) correspond to a particular class of the so-called Hahn polynomials (Szegö, 1959 [19]; Levit, 1967 [12]; Morrison, 1969 [15]; Chihara, 1978 [3]; Nikiforov, Suslov & Uvarov, 1991 [17]), denoted by $h_n^{\alpha,\beta}(x,N)$. This class is composed by the classical Chebychev polynomials of a discrete variable $t_n(x,N) = h_n^{0,0}(x,N)$, which where introduced by the Russian mathematician P. L. Chebyshev in the past century (see [2]). Until now, however, the study of the algebraic and spectral properties of the Chebyshev polynomials is a very interesting mathematical topic which receives much attention in the modern theory of special functions (Nikiforov, Suslov & Uvarov, 1991 [17]; Dette, 1995 [5]; Rakhmanov, 1996 [18]; Kuijlaars & Van Assche, 1997 [10]; Alvarez-Nodarse & Dehesa, 1998 [1]).

The purpose of this paper is to identify the *speech* polynomials as shifted Chebyshev polynomials. In doing so, we observe that some mathematical tools used in some speaker recognition methods could be considerably reduced (see e.g. the recurrence relation used in the Liu-Wang paper [11, Appendix A] for the orthogonal polynomials), what can imply a big reduction and simplification in the algorithms inherent to these methods.

The structure of the paper is as follows. In Section 2 some definitions and statements of the general theory of orthogonal polynomials are given. Then, in Section 3, the *speech* polynomials are identified as shifted Chebyshev polynomials. Finally, in Section 4, the spectrum of zeros of the speech polynomials as a whole is studied by the explicit determination of the moments-around-the-origin of the distribution of zeros of the speech polynomials.

2 Basic background on orthogonal polynomials.

In this Section we will describe some well known facts from the general theory of orthogonal polynomials and, specifically, from the Chebyshev polynomials of a discrete variable which will help us in the next Section to identify the orthogonal polynomials found in the Lie-Wang model [11].

Let $\mu(x)$ be a non-constant and non-decreasing function in $[a,b]$ (if any of a,b are $\pm\infty$ we require that $\mu(\pm\infty)$ should be finite). Let us define the scalar product of two real functions

f and g by the Stieltjes-Lebesgue integral

$$(f,g) = \int_a^b f(x)g(x)d\mu(x), \tag{2.1}$$

where we suppose that f, g are square integrable functions belongs to $L_2(\mu)$, i.e., $\int_a^b f^2(x)d\mu(x) < +\infty$.

In what follows f, g are continuous in $[a,b]$ with a, b finite real numbers. Two particular cases of special interest correspond to the ones when μ is absolutely continuous positive functions or a step function with jumps at a finite number of points $\{x_i\}_{i=1}^N$, $x_i \in [a,b]$, $i = 1, 2, ..., N$. In these two cases the scalar product (2.1) becomes

$$(f,g) = \int_a^b f(x)g(x)\rho(x)\,dx, \quad \text{and} \quad (f,g) = \sum_{i=1}^N f(x_i)g(x_i)\rho(x_i), \quad \rho(x) > 0, \quad \forall x \in [a,b], \tag{2.2}$$

respectively and ρ is said to be a continuous or discrete *weight function*, also respectively.

A system of functions f_1, f_2, ..., f_n in $L_2(\mu)$ is called an orthogonal system if $(f_i, f_j) = 0$ for all $i \neq j$. Obviously, orthogonal functions are necessarily linearly independent. In the case when μ has a finite number N of point of increase (like in the case of a discrete weight function mentioned above), n is necessarily finite: $n \leq N$.

Given a sequence of linearly independent functions in $L_2(\mu)$, it is always possible to obtain an orthogonal sequence. This procedure is called the orthogonalization or Gram-Schmidt process. The simplest set of continuous functions is the sequence of non-negative powers 1, x, x^2, ..., x^n, ... Since we assume that $[a,b]$ is bounded, then from this set we can derive an orthogonal set. In fact, if we denote by Δ_n the following determinant

$$\Delta_n = \begin{vmatrix} \mu_0 & \mu_1 & \cdots & \mu_n \\ \mu_1 & \mu_2 & \cdots & \mu_{n+1} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_n & \mu_{n+1} & \cdots & \mu_{2n} \end{vmatrix}, \tag{2.3}$$

where $\mu_k = \int_a^b x^k d\mu(x)$, $k = 0, 1, 2, ...$ are the moments associated to μ, then the Gram-Schmidt orthogonalization process leads us to the following set of orthogonal polynomials

$$p_n(x) = A_n \begin{vmatrix} \mu_0 & \mu_1 & \cdots & \mu_n \\ \mu_1 & \mu_2 & \cdots & \mu_{n+1} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{n-1} & \mu_n & \cdots & \mu_{2n-1} \\ 1 & x & \cdots & x^n \end{vmatrix} = a_n x^n + \cdots, \qquad n = 0, 1, 2, ..., \quad a_n \neq 0\,, \tag{2.4}$$

being A_n some constant. It is straightforward to see that $(p_k, p_n) = 0$, for all $k = 0, 1, ..., n$, i.e., the following theorem holds (see e.g. [3, 16, 19])

Theorem 1 *Given a distribution function μ with moments μ_k, $k = 0, 1, 2, ...$, there exists a uniquely determined up to a constant multiplicative factor sequence of orthogonal polynomials $\{p_n\}$, defined by (2.4), each of which have degree exactly equal n, providing that $\Delta_n > 0$ for all $n \geq 0$.*

Remark 1 In the case when ρ is a positive definite weight function, $\Delta_n > 0$ for all $n \geq 0$.

Remark 2 This theorem states that, given a distribution function μ with moments μ_k, then by the standard Gram-Schmidt orthogonalization process, we can construct one and only one sequence of polynomials orthogonal with respect to μ (i.e., a sequence such that $(p_n, p_m) = 0$ if $n \neq m$) provide that, for example, the leading coefficient a_n is fixed. The most standard normalizations for the orthogonal polynomials correspond to the two following choices: (i) $a_n = 1$, in which case one speaks about monic orthogonal polynomials, and (ii) $a_n = (p_n, p_n)^{-\frac{1}{2}}$, then one deals with orthonormal polynomials.

Since the polynomials p_n constitute a linearly independent set, then one can expand any polynomial π of degree m in the set $\{p_0, p_1, ..., p_m\}$. Using this fact and the the orthogonality property of the polynomials p_n one obtains

Theorem 2 *If $\{p_n\}_{n=0}^{\infty}$ is a monic orthogonal polynomial sequence with respect to a weight function $\rho(x)$, then the polynomials p_n satisfy a three-term recurrence relation of the form*

$$p_n(x) = (x - c_n)p_{n-1}(x) - \lambda_n p_{n-2}(x), \qquad p_{-1}(x) = 0,\ p_0(x) = 1, \quad n = 1, 2, 3, \dots \,, \tag{2.5}$$

where $\{c_n\}_{n=0}^{\infty}$ and $\{\lambda_n\}_{n=0}^{\infty}$ are given by

$$c_n = \frac{(xp_{n-1}, p_{n-1})}{(p_{n-1}, p_{n-1})}, \quad n \geq 1, \quad \textit{and} \quad \lambda_n = \frac{(xp_{n-1}, p_{n-2})}{(p_{n-2}, p_{n-2})} \ , \quad n \geq 2 \ ,$$

respectively.

In this paper we will deal with the classical discrete Chebyshev monic polynomials $t_n(x, N)$. These polynomials $t_n(x, N)$ are polynomials that satisfy an orthogonality relation of the form

$$\sum_{x=0}^{N-1} t_n(x, N)\, t_m(x, N) = \delta_{nm} \frac{n!^2 (N+n)!}{(2n+1)(N-n-1)!(n+1)_n^2}, \tag{2.6}$$

where δ_{nm} is the Kronecker symbol ($\delta_{nm} = 1$ if $n = m$ and 0 elsewhere) and $(a)_n = a(a+1)\cdots(a+n-1)$ denotes the Pochhammer symbol. That is, they are orthogonal with respect to a distribution function μ, which is a step function with N jumps at the points $x = 0, 1, ..., N-1$ but in this case, as we already pointed out (see e.g. [19, page 24]), they form a finite family of orthogonal polynomials.

These polynomials satisfy the three-term recurrence relation (2.5) with coefficients

$$c_n = \frac{N-1}{2}, \qquad \lambda_n = \frac{(n-1)^2[N^2 - (n-1)^2]}{4[4(n-1)^2 - 1]}. \tag{2.7}$$

Moreover, they form a very important special subclass of the Hahn polynomials $h_n^{\alpha,\beta}(x, N)$, [16, 17]: that with $\alpha = \beta = 0$.

3 Identification of the speech polynomials

In the Lee-Wang model [11, Section 3.1] a sequence of orthogonal polynomials $\phi_n^L(l)$ is introduced to regenerate a time sequence of $L-$length feature vectors. This family of polynomials satisfy an orthogonality relation [11, Appendix A, Eq. (48)]

$$\sum_{l=1}^{L} \phi_n^L(l) \phi_k^L(l) = 0, \quad n \neq k, \quad n, k = 0, 1, 2, ..., R. \tag{3.1}$$

Obviously, the above orthogonality relation corresponds to the discrete scalar product (2.2). Then, theorem 1 states that the polynomials $\phi_n^L(l)$ are uniquely determined up to a constant factor. Moreover, since the distribution function μ is a step function with L jumps at points $x = 1, 2, ..., L$, the family $\phi_n^L(l)$ is a finite family, i.e., in (3.1) $R \le L - 1$.

If we now compare the orthogonality relation (3.1) with (2.6), one can easily arrive to the conclusion that the speech polynomials $\phi_n^L(l)$ are proportional to the Chebyshev polynomials $t_n(l-1, L)$. In fact, making the change of variable $x \to l - 1$ in (2.6)

$$0 = \sum_{x=0}^{L-1} t_n(x, L)\, t_k(x, L) = \sum_{l=1}^{L} t_n(l-1, L)\, t_k(l-1, L), \quad n \ne k, \quad n, k = 0, 1, 2, ..., L-1,$$

we arrive to (3.1). Moreover, the square norm of the ϕ_n^L, denoted in [11, Eq. (38)] by Φ_n^L, has the explicit form

$$\sum_{l=1}^{L} \phi_n^L(l) \phi_n^L(l) = \frac{n!^2 (L+n)!}{(2n+1)(L-n-1)!(n+1)_n^2}.$$

For simplicity, let us consider the monic polynomials, i.e., the polynomials $\phi_n^L(l) = l^n + \cdots$. With this normalization, and using the the three-term recurrence relation for the Chebyshev polynomials (2.7), we obtain that the speech polynomials $\phi_n^L(l)$ satisfy a three-term recurrence relation of the form

$$\phi_n^L(l) = \left(l - \frac{L+1}{2}\right) \phi_{n-1}^L(l) - \frac{(n-1)^2[L^2 - (n-1)^2]}{4[4(n-1)^2 - 1]} \phi_{n-2}^L(l).$$

which is nothing else that the relation (50) in the Liu-Wang model [11]. Most important is to remark that the α and β values of this model [11] (see Eqs. (51) and (52) in [11]) reduce to

$$\alpha = 0, \qquad \beta = -\frac{(n-1)^2[L^2 - (n-1)^2]}{4[4(n-1)^2 - 1]}.$$

Obviously, from the properties of the Chebyshev polynomials [5, 17, 19], one can obtain a lot of properties for the $\phi_n^L(l)$. In fact

1. Second order difference equation

$$(l-1)(L-l+1) \triangle \nabla \phi_n^L(l) + (L+1-2l) \triangle \phi_n^L(l) + n(n+1) \phi_n^L(l) = 0,$$
$$\triangle f(x) = f(x+1) - f(x), \quad \text{and} \quad \nabla f(x) = f(x) - f(x-1).$$

2. Explicit formula

$$\phi_n^L(l) = \frac{(-1)^n}{(n+1)_n} \sum_{k=0}^{n} (-1)^k \frac{n!}{k!(n-k)!} \frac{\Gamma(L-l+k+1)\Gamma(n+l-k)}{\Gamma(L-n-l+k+1)\Gamma(l-k)},$$

$$\phi_n^L(1) = (-1)^n \frac{n!(L-1)!}{(n+1)_n (L-n-1)!}, \qquad \phi_n^L(L) = \frac{n!(L-1)!}{(n+1)_n (L-n-1)!}.$$

3. Hypergeometric representation

$$\phi_n^L(l) = \frac{(1-L)_n n!}{(n+1)_n} \, {}_3\mathrm{F}_2\left(\begin{matrix} 1-l\,,\, n+1\,,\, -n \\ 1-L\,,\, 1 \end{matrix} \,\middle|\, 1 \right),$$

where the hypergeometric function ${}_p\mathrm{F}_q$ is defined by

$${}_p\mathrm{F}_q\left(\begin{matrix} a_1, a_2, ..., a_p \\ b_1, b_2, ..., b_q \end{matrix} \,\middle|\, x \right) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_p)_k}{(b_1)_k (b_2)_k \cdots (b_q)_k} \frac{x^k}{k!}.$$

4. Generating function

$$\sum_{n=0}^{L-1} \frac{(n+1)_n}{(n!)^2} \phi_n^L(l) z^n = {}_1\mathrm{F}_1\left(\begin{matrix} l-N \\ 1 \end{matrix} \Big| z \right) {}_1\mathrm{F}_1\left(\begin{matrix} 1-l \\ 1 \end{matrix} \Big| z \right), \quad l = 1,2,...,L.$$

5. Symmetry property

$$\phi_n^L(L-l+1) = (-1)^n \phi_n^L(l).$$

6. Dette inequality

$$|\phi_n^L(l)| \le \frac{(L-n)_n}{(n+1)_n}.$$

From the above expressions we obtain the following expression for the five first ϕ_n^L

$$\phi_0^L(l) = 1, \qquad \phi_1^L(l) = -\tfrac{L+1}{2} + l, \quad \phi_2^L(l) = \tfrac{(1+L)(2+L)}{6} - (L+1)\, l + l^2\,,$$

$$\phi_3^L(l) = -\tfrac{((1+L)(2+L)(3+L)}{20} + \tfrac{\left(11+15L+6L^2\right)}{10}\, l - \tfrac{3(1+L)}{2}\, l^2 + l^3\,,$$

$$\phi_4^L(l) = \tfrac{(1+L)(2+L)(3+L)(4+L)}{70} - \tfrac{(1+L)\left(10+7L+2L^2\right)}{7}\, l + \tfrac{\left(17+21L+9L^2\right)}{7}\, l^2 - 2\,(1+L)\, l^3 + l^4\,,$$

$$\phi_5^L(l) = -\tfrac{(1+L)(2+L)(3+L)(4+L)(5+L)}{252} + \tfrac{\left(274+525L+365L^2+105L^3+15L^4\right)}{126}\, l - \tfrac{5(1+L)\left(5+3L+L^2\right)}{6}\, l^2 +$$

$$+\tfrac{5\left(8+9L+4L^2\right)}{9}\, l^3 - \tfrac{5(1+L)}{2}\, l^4 + l^5\,.$$

The graphical representation of these five polynomials are shown in 1a and 1b for $L = 7$ and $L = 12$, respectively.

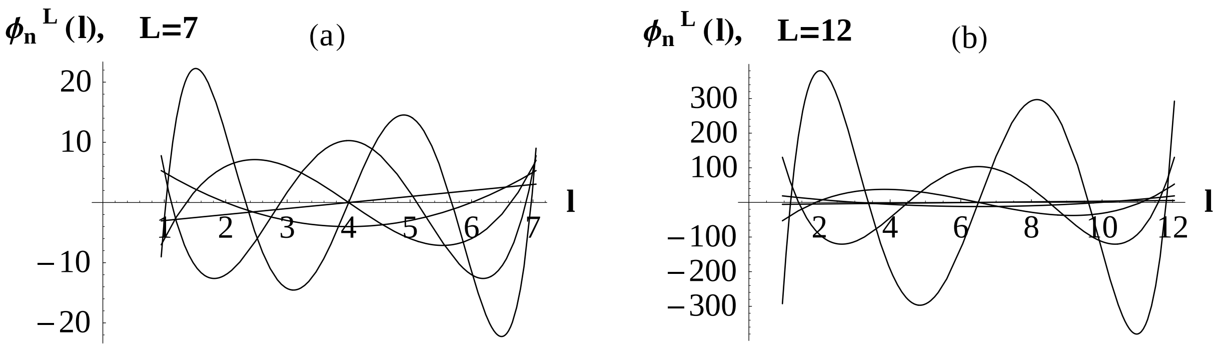

Figure 1: The speech polynomials ϕ_n^L, $n = 1,2,3,4,5$ and $L = 7$ and 12

4 Spectral moments of the speech polynomials.

Some important spectral characteristics of the speech polynomials are the moments of their zeros, which are defined by

$$\mu_0 = 1, \quad \mu_m'^{(n)} = \frac{1}{n} \sum_{k=1}^{n} x_{k,n}^m, \quad m = 1,2,...,L-1, \quad n \le L-1, \tag{4.2}$$

where $x_{k,n}$, $k = 1, 2, ..., n$ denotes the zeros of the polynomial ϕ_n^L. To obtain these quantities we can use the method given in [1]. In this way we have

$$\mu_m'^{(n)} = \frac{1}{n} \sum_{(m)} F(r_1', r_1, ..., r_j, r_{j+1}') \sum_{i=1}^{n-s} \prod_{k=1}^{j+1} \left[\frac{(L+1)}{2}\right]^{r_k'} \prod_{k=1}^{j} \left[\frac{(i+k-1)^2[L^2-(i+k-1)^2]}{4[4(2i+2k-1)^2-1]}\right]^{r_k},$$

where s denotes the number of non-vanishing r_i which are involved in each partition of m. The first summation runs over all partitions $(r_1', r_1, ..., r_{j+1}')$ of the number m such that

1. $R' + 2R = m$, where R and R' denote the sums $R = \sum_{i=1}^{[\frac{m}{2}]} r_i$ and $R' = \sum_{i=1}^{[\frac{m}{2}]-1} r_i'$, or

$$\sum_{i=1}^{[\frac{m}{2}]-1} r_i' + 2 \sum_{i=1}^{[\frac{m}{2}]} r_i = m\,. \tag{4.3}$$

2. If $r_i = 0,\ 1 < i < [\frac{m}{2}]$, then $r_k = r_k' = 0$ for each $k > i$ and
3. $[\frac{m}{2}] = \frac{m}{2}$ or $[\frac{m}{2}] = \frac{m-1}{2}$ for m even or odd, respectively.

The factorial coefficient F is defined by

$$F(r_1', r_1, r_2', ..., r_{p-1}', r_{p-1}, r_p') =$$

$$= m \frac{(r_1' + r_1 - 1)!}{r_1'! r_1!} \left[\prod_{i=2}^{p-1} \frac{(r_{i-1} + r_i' + r_i - 1)!}{(r_{i-1} - 1)!\, r_i!\, r_i'!}\right] \frac{(r_{p-1} + r_p' - 1)!}{(r_{p-1} - 1)!\, r_p'!}, \tag{4.4}$$

with the convention $r_0 = r_p = 1$. For the evaluation of these coefficients, we must take into account the following convention

$$F(r_1', r_1, r_2', r_2 ..., r_{p-1}', 0, 0) = F(r_1', r_1, r_2', r_2 ..., r_{p-1}').$$

Then, the first few spectral moments of ϕ_n^Ls have the simpler expressions

$$\mu_1'^{(n)} = \frac{L+1}{2}, \quad \mu_2'^{(n)} = \frac{(1 + 3L(1+L)) + (2+3L)^2 n + 2n^2 - n^3}{24n - 12}$$

$$\mu_3'^{(n)} = \frac{(1+L)\left(-2L - 4L^2 + 4Ln + 5L^2 n + 2n^2 - n^3\right)}{16n - 8}.$$

These quantities give different dispersion measures of the distribution of zeros of the speech polynomials. The centroid of the distribution is $\mu_1'^{(n)}$, and the variance σ^2 is equal to

$$\sigma^2 = \mu_2'^{(n)} - (\mu_1'^{(n)})^2 = \frac{(n-1)\ \left(3L^2 + n - n^2 - 1\right)}{24n - 12}.$$

Also, it turns out that the skewness γ_1 vanishes and the excess or kurtosis is positive. The former is a straightforward consequence of the symmetric nature of the distribution while the latter indicates that the distribution of the zeros of the speech polynomials are sharper around the centroid than a Gaussian distribution of the same variance.

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