Curso intensivo y Workshop de Física Matemática

Una aproximación geométrica a los sistemas dinámicos (8 horas) Profs. J. Galán y E. Freire

El curso es una introducción a la teoría de los sistemas dinámicos con ejemplos desde el punto de vista de las Matemáticas y la Física.

En particular, se abordará la teoría cualitativa o geométrica de las ecuaciones diferenciales no lineales y de los sistemas discretos. Durante el curso se pondrá énfasis en aquellos aspectos que tienen interés en las aplicaciones (estabilidad, atractores, continuación y bifurcaciones) así como en la utilidad de los resultados teóricos en el modelado y simulación de sistemas concretos. Todo el enfoque teórico irá acompañado del estudio de técnicas computacionales diversas (tanto simbólicas como numéricas) y su aplicación a distintos problemas prácticos.

Temario:

1. Introducción.

2. Bifurcaciones elementales.

3. Sistemas dinámicos en la mecánica.

4. Herramientas numéricas para estudiar sistemas dinámicos; continuación y simulación.

Bibliografía básica:

Y. Kuznetsov, Elements of Applied Bifurcation Theory, Springer 2004.

S. Strogatz, Nonlinear Dynamics and Chaos, Addison Wesley 2000.

Charlas de aplicaciones asociadas al curso:

Charlas 1 y 2: F. Guillen y J.M. Romero Enrique, Liquid Crystal and Phase-Field models are related by Mathematics. Martes 7 de julio 11:30

Charlas 3 y 4: J.E. García-Ramos, Quantum Phase Transitions in simple Two-Level Systems and the Catastrophe Theory. Miércoles 8 de julio 9:00

De la mecánica cuántica a las funciones especiales y las representaciones de grupos y álgebras de Lie

Introducción a la teoría de algebras y grupos de Lie. Clasificación y descripción de sus representaciones irreducibles. (8 horas) Prof. E. Briand

Temario:

1. Grupos. Grupos en general, grupos de transformaciones. Grupos de Lie. Grupos de Lie de matrices. Álgebra de Lie asociada a un grupo de Lie. Álgebras de Lie. Dimensión.

2. Representaciones de grupos de Lie y álgebras de Lie. Representaciones de grupos y de álgebras de Lie. Representaciones irreducibles. Representaciones en espacios de tensores. Representaciones como operadores diferenciales. Bosones, fermiones. Clasificación de las representaciones. El caso de SU(n): descripción de las representaciones irreducibles.

3. Representaciones: bases. Bases canónicas de representaciones irreducibles, cadenas de grupos y álgebras. Restricción a un subgrupo. Productos tensoriales de representaciones. Coeficientes de Clebsch-Gordan.

4. Representaciones modulo isomorfismo. Caracteres. Funciones simétricas y combinatoria. Regla de Littlewood-Richardson. Coeficientes de Littlewood-Richardson / Serie de Clebsch-Gordan.

Bibliografía básica:

Robert Gilmore, Lie groups, physics, and geometry: an introduction for physicists, engineers and chemists. Cambridge University Press, 2008

William Fulton, Joe Harris, Representation theory: a first course. Springer-Verlag, 1991

Introducción a la Mecánica cuántica para matemáticos (y no sólo) y a la teoría de funciones especiales y polinomios ortogonales. (8 horas) Profs. R. Alvarez-Nodarse y A.J. Durán

Temario:

1. Breve introducción histórica: de las nubecillas de Lord Kelvin a la revolución cuántica

2. Espacios de Hilbert y operadores autoadjuntos. Las transformaciones de simetría.

3. Los postulados de la Mecánica cuántica y sus consecuencias.

4. La ecuación hipergeométrica y los polinomios ortogonales.

5. Resolviendo la ecuación de Schrödinger: el método de Nikiforov y Uvarov.

Bibliografía básica:

L. Debnath y P. Mikusinsk. Introduction to Hilbert spaces with applications, Elsevier Academic Press, 2005

A. F. Nikiforov y V. B. Uvarov, Special Functions of Mathematical Physics. Birkhäuser Verlag 1988.

Charlas de aplicaciones asociadas al curso:

Lecture 1. F. Iachello. Nuclear Physics: The Interacting Boson Model. Martes 7 de julio 9:00

Lecture 2. F. Iachello. Particle Physics: The Quark Model. Martes 7 de julio 10:00

Lecture 3. F. Pérez Bernal. Molecular Physics: The Vibron Model Miércoles 8 de julio 11:30

Lecture 4. J.M. Arias Carrasco. Discretising the continuum in simple 1D systems using Orthogonal Polynomials Miércoles 8 de julio 12:30

Horario de las distintas sesiones de los cursos

	L	M	X	J	V
9:00-11:00	MC1	TG2	PO	SD2	TG4
11:00-11:30	Coffee	Coffee	Coffee	Coffee	Coffee
11:30-13:30	TG1	MC2	SD1	TG3	SD3
16:00-18:00		TUTORIAS		TUTORIAS	

	L	M	X
9:00-11:00	Sch. Eq.	Iachello	García-Ramos
11:00-11:30	Coffee	Coffee	Coffee
11:30-13:30	SD4	Guillen y Romero	Arias y Pérez
16:00-18:00		Charlas alumnos	

MC: Introducción a la Mecánica Cuántica

PO: Introducción a los polinomios ortogonales

TG: Introducción a la Teoría de Representación de Grupos

Sch. Eq. Resolviendo la Ecuación de Schrödinger

SD: Introducción a los sistemas dinámicos

Charlas de alumnos del curso

- Alberto Arenas Gómez

Título: Weighted inequalities for the intertwining operator for radial functions on the sphere.

In 1958 Stein and Weiss [2] proved that, under suitable conditions, the fractional integral operator for the Laplacian satisfies certain inequality on the spaces L^d. This result can not be improved for general functions in L^q. However, De Napoli, Drelichman and Duran [1] extended the range of admissible power weights with f a radial function. In this talk, we prove the analogous theorem of De Napoli, Drelichman and Duran result in the setting of the d-dimensional sphere S^(d-1). In fact we will prove an inequality for functions which are invariant under the action of the group SO(S^(d-1)).

[1] P. L. De Napoli, I. Drelichman, and R. G. Dur\'an: On weighted inequalities for fractional integrals of radial funcions, Illinois J. Math. 55 (2) (2011), 575-587.

[2] E. M. Stein and G. Weiss: Fractional integrals on n-dimensional Euclidean space, J. Math. Mech. 7(4) (1958), 503-514.

- Yolanda Cabrera Casado

Título: Álgebras de evolución.

En los 1920 y 1930, se incorporó un nuevo concepto al campo de las matemáticas, fruto de la interacción entre la genética y el álgebra. Mendel y otros genetistas lograron dar una formulación matemática de las llamadas leyes de Mendel, interpretando la multiplicación como la reproducción sexual. No obstante, a comienzos del siglo XX se descubrieron algunos organismos cuya herencia genética no sigue dichas leyes. Este hecho introdujo un nuevo concepto, las álgebras de evolución. En esta charla presentaremos algunos resultados relacionados con la caracterización de las álgebras de evolución de dimensión 2 y 3 así como la descomposición de las dichas álgebras en suma directa, utilizándose para ello la teoría de grafos.

- Alejandra Sarina Córdova Martínez

Título: Graduaciones sobre álgebras de Lie

Las álgebras de Lie admiten graduaciones interesantes que ayudan a conocer propiedades de su estructura. En esta charla definiré lo que es un álgebra de Lie, una graduación y algunos casos especiales de graduaciones, además se construirá el grupo universal de una graduación.

- Marina Esteban Pérez

Título: Reducción de sistemas tridimensionales lineales a trozos con dinámica lenta-rápida a sistemas planos con histéresis.

Consideramos sistemas tridimensionales lineales a trozos, con simetría respecto al origen y dinámica lenta-rápida. Asumiendo despreciables los tiempos de transición de la dinámica rápida, llegamos a un sistema plano donde la única no linealidad es de tipo histéresis. Partiendo de las situaciones más simples, determinamos la posible existencia de órbitas periódicas mediante análisis de bifurcaciones.

- Javier López de la Cruz

Título: Dynamics of a non-autonomous chemostat

Chemostat refers to a laboratory device used for growing microorganisms in a cultured environment and has been regarded as an idealization of nature to study competition modelling in mathematical biology. The simplest form of chemostat model assumes that the availability of nutrient and its supply rate are both fixed. In addition, the tendency of microorganisms to adhere to surfaces is neglected by assuming the flow rate is fast enough. Some basic concepts related to non-autonomous dynamical systems are explained in this lecture, illustrated with several interesting models, developed in the study of my PhD thesis.

- Laura Moschini

Título: Description of quantum-mechanical collisions in a one-dimensional toy model

A simple one-dimensional model to describe collisions between quantum systems with internal degrees of freedom is presented. The time evolution is given by solving the time dependent Schroedinger equation. These kind of models are developed to test the usual theoretical methods based on approximations; in this case we simulate the direct reactions between two nuclei and in particular we present the comparison between the "exact" solution and approximate approaches for the inelastic scattering problem.

- Carlos A. Plata

Título: Study of the unfolding pathway through a simple model of biomolecules

The dependence of the unfolding pathway of proteins on the pulling speed is investigated. This is done by introducing a simple one-dimensional chain comprising N units, with different characteristic bistable free energies. These units can be either folded or unfolded. The system is pulled by applying a force to the last unit of the chain. The force is such that the total length is controlled. Under this pulling experiment the units unfold following a preferred sequence. We show that the unfolding sequence strongly depends on the pulling velocity v_p as is observed in recent simulations. In the simplest situation, there appears a critical pulling speed v_c: For pulling speeds $v_p < v_c$, the weakest unit unfolds first, whereas for $v_p > v_c$ it is the pulled unit that unfolds first. The description of the dynamics is given by the corresponding system of Langevin equations. We find a quite accurate expression for the critical velocity by means of a perturbative expansion of the solutions in the asymmetry of the free energies and the pulling speed. We do so in the deterministic limit (nule thermal fluctuation) and perfect control over the total length.

- Adrián Rodrigo Escudero

Título: Clasificación de las graduaciones de división en álgebras simples reales de dimensión finita

Los teoremas de Wedderburn-Artin (1907, 1927) y de Frobenius (1877) clasifican las álgebras simples reales de dimensión finita, que resultan ser las álgebras de matrices sobre los reales, los complejos y los cuaternios. Elduque y Kochetov prueban en [1] un equivalente del teorema de Wedderburn-Artin para graduaciones. En esta charla hablaré sobre mi trabajo de los últimos meses [2], que consiste en clasificar, salvo isomorfismo y salvo equivalencia, las graduaciones de división (sobre grupos abelianos) en álgebras simples reales de dimensión finita, estableciendo un análogo al teorema de Frobenius y completando la clasificación.

[1] A. Elduque and M. Kochetov, Gradings on simple Lie algebras, Mathematical Surveys and Monographs, 189, Amer. Math. Soc., Providence, RI, 2013.

[2] A. Rodrigo-Escudero, Classification of division gradings on finite-dimensional simple real algebras, arXiv:1506.01552 [math.RA].

ABSTRACTS of contributions to the WORKSHOP

Conferencia inaugural: Francesco Iachello (Yale University)

Title: The role of symmetry in nuclear physics

Both continuous and discrete symmetries have played an important role in nuclear physics since the introduction of isospin symmetry by Heisenberg in 1932. In the first part of this presentation, the dynamic symmetries of the Interacting Boson Model (IBM), U(5), SU(3) and SO(6), and of the Proton-Neutron Interacting Boson Model (IBM-2), $U_{\pi\nu}(5)$, $SU_{\pi\nu}(3)$, $SO_{\pi\nu}(6)$, $SU_{\pi\nu}$*(3), will be discussed as examples of continuous symmetries in nuclear physics. In the second part, the symmetries of the algebraic cluster model (ACM), Z_2, D_3 and T_d, will be discussed as examples of discrete symmetries in nuclear physics. Experimental evidence for both continuous and discrete symmetries will be presented.

Conferencia de clausura: Peter Hänggi (University of Augsburg)

Title: On the meaning of temperature in different ensembles

Let us elaborate on the notion of thermodynamic entropy S (Clausius 1865) and its consequences. Most importantly, *the temperature is a derived quantity*: It is given as the thermodynamic force of the thermodynamic state function, known as the entropy S. The absolute temperature T then obeys:

$1/T = \partial S/\partial E$,

wherein E denotes the internal thermodynamic energy state function. As such the inverse absolute temperature provides the integrating factor for the Second Law of thermodynamics, $dS = \delta Q^{rev}/T$, where δQ^{rev} refers to the reversible, quasi-static heat exchange. Statistical mechanics is supposed to describe this entropy function from first principles. We recall here that the most fundamental statistical equilibrium ensemble is the micro-canonical ensemble (MCE). This is so because the canonical, the grand canonical ensemble, and all others, follow from this MCE.

In this context Josiah Willard Gibbs introduced (among others) two thermodynamic entropy expressions for the (MCE). A first one (i) is known as *volume* entropy, termed here the `Gibbs entropy' S_G, which in modern notation reads: $S_G(E, \boldsymbol{\lambda}) = k_B \ln \Omega(E, \boldsymbol{\lambda})$, with $\boldsymbol{\lambda}$ denoting the set of external control parameters, such as the available volume, magnetic field, etc.. Here $\Omega(E, \boldsymbol{\lambda})$ is the integrated, non-negative valued density of states (DoS) over all energies E' not exceeding E. The DoS is thus given by: $\omega(E', \boldsymbol{\lambda}) = \partial\, \Omega(E', \boldsymbol{\lambda})/\partial E'$. Gibbs also discussed a second entropy expression (ii) that is referred to as *surface* entropy S_B (nowadays commonly known also as the Boltzmann entropy), reading $S_B(E, \boldsymbol{\lambda}) = k_B \ln [\varepsilon\, \omega(E, \boldsymbol{\lambda})]$, with ε being some small energy constant so that the argument of the logarithm becomes dimensionless. As recently shown with Ref. **[1]**, the consistency of an entropy function S with the thermodynamics, that is to say with S obeying the celebrated **0-th**, **1-st** and **2-nd** thermodynamic Laws, then singles out the Gibbs-entropy **[1].**

I point out shortcomings that relate to the thermodynamics of finite systems when sticking to the (Boltzmann)-surface entropy **[2]**. Most of all, the uncritical use of Boltzmann entropy for an isolated micro-canonical system can formally yield negative values of absolute temperatures in systems possessing an upper bound in energy, such as it is the case, for example, with spin systems. This is not only physically incorrect for the concept of an *absolute* temperature, but also would violate thermodynamic stability if the system is brought into (weak) contact with an omnipresent

sort of environment of radiation source or otherwise with no upper bound in energy **[1,3].** One finds that the two corresponding thermodynamic temperatures T_G and T_B are not the same. It is only for the majority of physical systems that can be classified as `normal', i.e., with their DoS increasing with increasing total energy (such as in ideal gases) that the two temperatures practically become indistinguishable for large particle numbers. Clearly, the Gibbs absolute temperature T_G can never be negative-valued, this being in distinct contrast to the Boltzmann temperature $T_{B,}$ which may assume negative values. Nonvanishing differences, even in the thermodynamic limit, can occur as well for other thermodynamic forces, such as for example for the magnetization in large spin systems **[3]**. The uncritical use of the Boltzmann entropy S_B may therefore indicate fictitious, nonphysical features.

I shall discuss next canonical entropy S_{can} when describing quantum systems that interact *strongly* with an environment. Then, the entropy and the canonical specific heat can -- against common wisdom known from weakly coupled systems -- assume negative values away from absolute zero temperature T = 0 **[4,5].**

[1] S. Hilbert, P. Hänggi, and J. Dunkel, *Thermodynamic Laws in Isolated Systems*, Phys. Rev. E **90**, 062116 (2014). (editor's suggestion)
[2] J. Dunkel and S. Hilbert, *Phase transitions in small systems: microcanonical vs. canonical ensembles*, Physica A **370**, 390 (2006).
[3] M. Campisi, *Construction of microcanonical entropy on thermodynamic pillars* arXiv:1411.2425v3.
[4] P. Hänggi, G.L. Ingold, and P. Talkner, *Finite quantum dissipation: the challenge of obtaining specific heat*, New J. Phys. **10**, 115008 (2008); ibid, *Specific heat anomalies of open quantum systems*, Phys. Rev. E **79**, 061105 (2009).
[5] M. Campisi, P. Talkner, and P. Hänggi, *Thermodynamics and fluctuation theorems for a strongly coupled open quantum system: an exactly solvable case*, J. Phys. A: Math. Theor. (Fast Track) **42**, 392002 (2009).

See also the two links:

(1) "**What is Temperature**"
http://www.physik.uni-augsburg.de/theo1/hanggi/Temperature.html

(2) "**(Quantum)-FluctuationTheorems**"
http://www.physik.uni-augsburg.de/theo1/hanggi/Fluctuation.html

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Conferencias de 60 min:

Florentino Borondo (UAM e ICMAT): TBA

Title: Classic and Quantum Chaos with applications to molecular vibrations

Paco Gancedo (Universidad de Sevilla)

Title: Mathematics & Fluids

The mathematical analysis of fluid mechanics models is a classical topic of research since Euler's 1757 paper, where the equation of an ideal flow was first derived. It is a current area of mathematical research of fundamental interest in particular due to its physical relevance and broad applicability. The purpose of this talk is to show well established models and to present basic and

important open questions. This will be carried out by showing some recent results of the speaker.

Rosario González-Feréz (Universidad de Granada) TBA

Title: Laser orientation and alignment of molecules

We present a theoretical study of the impact of combined electric and non-resonant laser fields on the rotational dynamics of polar molecules. We assume that the electronic and vibrational motions of the molecules are not affected by the external fields, and analyze the rotational motion using the rigid rotor approach. In addition, we assume that the rotational period of the molecule is much smaller than the laser pulse duration, and work within this adiabatic approximation using a constant pulse envolope for the laser field. Then, we solve the time-independent Schröedinger equation by means of a basis set expansion in terms of the Legendre polynomials, spherical Harmonics or Wigner matrices. We discuss the field-dressed rotational motion in terms of the alignment and orientation along the field axes. We refine our physical model and take into account the time profile of the non-resonant laser pulse. Thus, we solve the time-dependent Schröedinger equation by using the split-operator method. We compare our theoretical predictions to the results of the mixed-field-orientation experiments. The experimental results are rationalized in terms of non-adiabatic phenomena due to the field-induced couplings between states. We demostrate that the adiabatic criteria "the laser pulse duration being longer than the molecular rotational period" is not correct. Based on our numerical results, we provide a new definition of this adiabatic limit and its physical interpretation.

Mercedes Rosas (Universidad de Sevilla)

Title: The Kronecker coefficients

We will introduce the Kronecker coefficients, talk about their interpretation in terms of the representation theory of the symmetric group and the general linear group, and give some applications to quantum information theory.

Conferencias 30 min:

Renato Alvarez-Nodarse (Universidad de Sevilla)

Title: Can the symmetry fix the shape of certain phenomena in non-linear Physics?

Dynamical systems often contain oscillatory forces or depend on periodic potentials. Time or space periodicity is reflected in the properties of these systems through a dependence on the parameters of their periodic terms. Is a matter of fact that the response of competely different kind of systems regardless of whether they are classical or quantum, stochastic or deterministic, dissipative or nondissipative, etc. have the same functional dependence to the parameters of the oscillatory forces. We will show that this is a consequence of simple symmetry considerations determine.

Jorge Galán Vioque (Universidad de Sevilla)

Title: Continuation of periodic orbits in conservative dynamical systems and its implications in Quantum mechanics

We illustrate the important role of the families of periodic orbits in understanding the global behaviour of a dynamical system and its implications in the quantum dynamical counterpart.

José Enrique García-Ramos (Universidad de Huelva)

Title: Quantum Phase Transitions in Coupled Systems

Two-fluid algebraic models are of great interest in the context of quantum phase transition in nuclei. We intend to study the phase diagram of a two-fluid Lipkin model that resembles a nuclear proton-neutron interacting boson model Hamiltonian. We study the mean-field energy surface of a consistent-Q like two-fluid Lipking Hamiltonian and we compare with exact results coming from a direct diagonalization. We obtain the phase diagram of the model, identifying the order of the different phase-transition lines and surfaces using a Catastrophe Theory analysis. We found that there are two first order surfaces, one separating the spherical and the deformed shape, while the other separating the ``oblate" and the ``prolate" phases. We also found a second order line finishing in a third order phase transition point.

Antonio Moro (Universidad de Sevilla)

Title: Stabilization of the Numerov method for solving coupled differential equations

The quantum-mechanical description of a scattering problem with internal degrees of freedom usually requires the solution of a system of coupled-differential equations, one for each configuration of the projectile, the target, and their relative motion (referred to as "channel"). For a system of N equations, N independent solutions can be found in general. The physical solution will correspond to a definite combination of these solutions, corresponding to a scattering problem, namely, an incident plane (or Coulomb wave) in some initial (incident) state, and outgoing spherical waves in all channels. The usual procedure is to integrate the system of equations N times (with N different initial conditions), starting at the origin (r=0) and up to a sufficiently large distance, for which all the coupling potentials have already vanished, and the analytical solution is known. A linear combination is then constructed and the coefficients are determined by equating this linear combination to the known asymptotic behaviour of the physical solution.

A common problem that arises in this method is the loss of accuracy due to the tendency of the solution vectors to become nearly linearly dependent during the integration through a classically forbidden region as an effect of round-off errors. The problem is aggravated when channels with very different classical turning points are present. Inside the classically forbidden region, the solutions are very small. When the integration goes through one of these turning points, the associated solution will typically grow by several orders of magnitude, becoming much larger than the other solutions. In this situation, even the smallest "contamination" of a slowly growing solution can result in a complete loss of identity of the original solution.

In this contribution, we present a simple stabilization procedure that is aimed at palliating this effect. The method is based on a re-orthogonalization procedure of the solutions, which is achieved by means of the well-known QR decomposition. Solutions vectors are arranged in the form of a square NxN matrix, Y, in which each column represents one of the N independent solutions. Linear independence of the solutions imply that the different columns must be mutually orthogonal. The afore-mentioned loss of linear independence means that some of these columns will not be mutually orthogonal any more. By means of the QR decomposition of the Y matrix (Y = Q R), one is able to construct a new set of solutions (Q) which, by construction, will be orthogonal. If the wavefunctions are required, the same transformation (given by the inverse of the triangular matrix R) must be apply backwards to the previous solutions. The method will be illustrated in some practical cases.

Pedro Pérez-Fernández (Universidad de Sevilla)

Title: Quantum Phase transitions in a two-site Bose-Hubbard with a Feshbach resonance

In this talk the Bose-Hubbard with a Feschbach resonance is presented. We explore quantum phase transitions in this model and we perform a beyond mean field study in order to get corrections to the mean field approach. Analytical results for the ground state energy and the first gaps are presented. The computed corrections are compared with numerical calculations. Besides, we analyse the onset of chaos in this model and its relationship with the observation of an excited state quantum phase transition.

Manuela Rodríguez-Gallardo (Universidad de Sevilla):

Title: Continuum treatment in weakly-bound systems of interest in Physics

In order to study weakly-bound systems, such as halo nuclei, it is essential to take into account the unbound states of the system. Since these unbound states form a continuum of energies, their inclusion in certain calculations requires the introduction of a discretization method, i.e., the representation of the continuum by a finite and discrete basis. Methods based on orthogonal polynomials are a useful tool to discretize the continuum of such systems. Examples for two- and three-body systems will be discussed.

	Thursday 9th	Friday 10th
9:00-9:30	Opening	
9:30-10:30	F. Iachello	F. Gancedo
10:30-11:00	A. Moro	R. Alvarez-Nodarse
11:00-11:30	Coffee Break	Coffe Break
11:30-12:30	R. Gonzalez Ferez	M. Rosas
12:30-13:00	M. Gallardo	
13:30-14:00	J. Galán	P. Hanngi
14:00-15:30	Lunch	
16:00-17:00	T. Borondo	
17:00-17:30	P. Perez Fernadez	
17:30-18:00	J.E. Garcia Ramos	