THE ROLE OF SYMMETRY IN NUCLEAR PHYSICS

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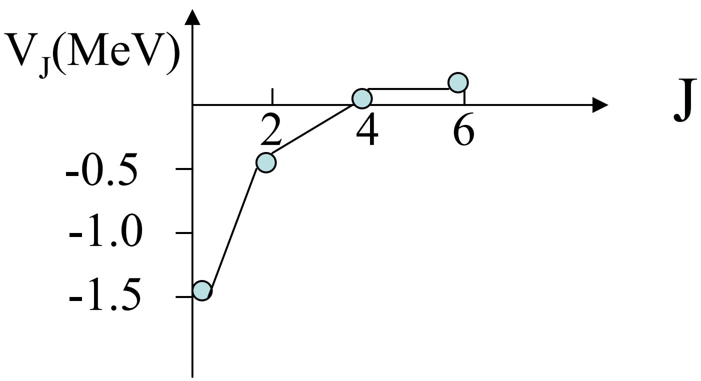
ROLE OF CONTINUOUS SYMMETRIES IN NUCLEAR PHYSICS

Early work:

1932	Heisenberg	$SU_T(2)$
1937	Wigner	$SU(4) \supset SU_T(2) \otimes SU_S(2)$
1940	Racah	$U(2\lambda+1)$, $U(2j+1)$
1958	Elliott	$SU(3)$

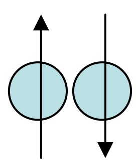
CONTINUOUS SYMMETRIES: THE INTERACTING BOSON MODEL U(6)

Constituents. The nucleus: protons and neutrons with strong interaction. Properties of the strong effective interaction: monopole and quadrupole pairing

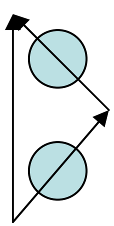


Even-even nuclei composed of nucleon pairs treated as bosons

⇒ The interacting boson model



J=0 S-pairs



J=2 D-pairs

Building blocks: nucleon pairs with J=0 and J=2 treated as bosons, s, d_μ ($\mu=0,\pm1,\pm2$).

Algebraic structure (spectrum generating algebra SGA)

$$G_{\alpha\beta} = b_\alpha^\dagger b_\beta \qquad \alpha,\beta=1,\ldots,6 \quad \Rightarrow \quad U(6)$$

$$H = E_0 + \sum_\alpha \varepsilon_\alpha G_{\alpha\alpha} + \frac{1}{2}\sum_{\alpha\alpha'\beta\beta'} u_{\alpha\alpha'\beta\beta'} G_{\alpha\alpha'} G_{\beta\beta'} + \ldots$$

Basis B: totally symmetric representations of U(6)

$$\left|N\right\rangle \equiv \underbrace{\square\square\ldots\square}_{\text{N-times}}$$

Breaking of U(6) into subalgebras (classification scheme) ¶

$$U(6) \supset U(5) \supset SO(5) \supset SO(3) \supset SO(2) \qquad \text{(I)}$$

$$U(6) \supset SU(3) \supset SO(3) \supset SO(2) \qquad \text{(II)}$$

$$U(6) \supset SO(6) \supset SO(3) \supset SO(2) \qquad \text{(III)}$$

Dynamic symmetries: situations in which the Hamiltonian H is a function only of Casimir operators of a chain $g \supset g' \supset g'' \supset \ldots$

$$H = \alpha C(g) + \alpha' C(g') + \alpha'' C(g) + \ldots$$

All properties of the system can be obtained in explicit analytic form in terms of quantum numbers characterizing the representations: Simplicity in complexity

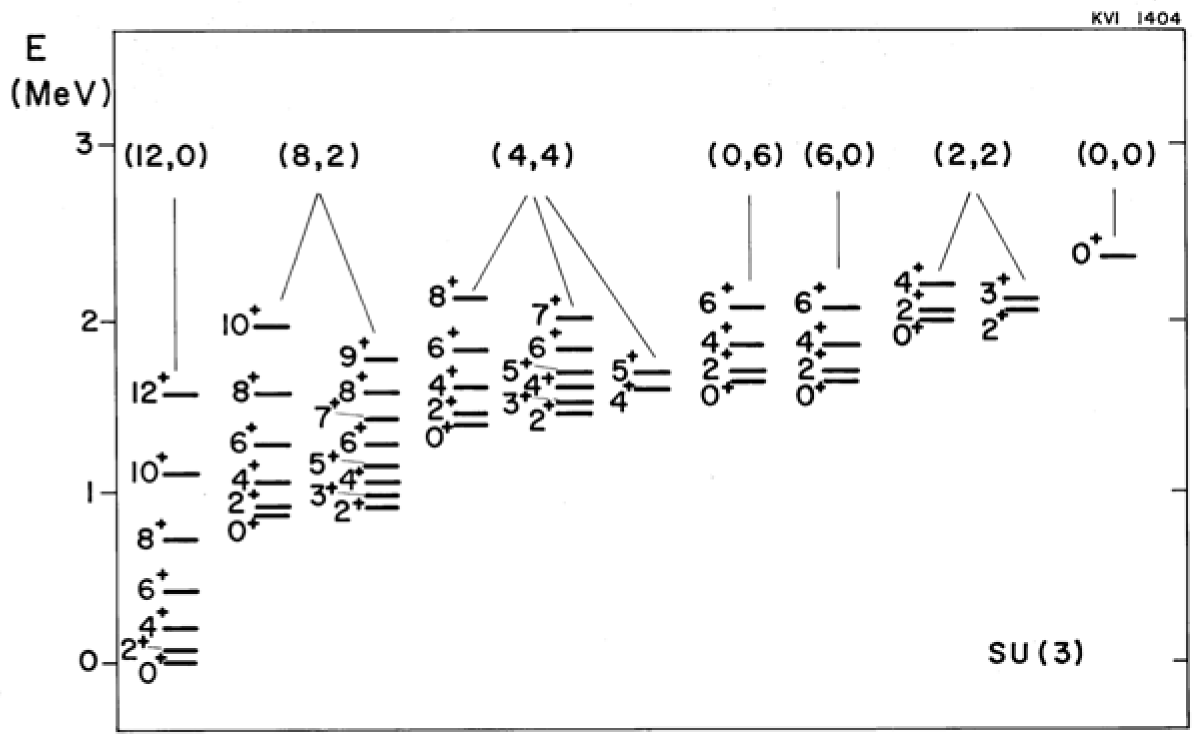
¶ A. Arima and F. Iachello, Ann. Phys. (N.Y.) 99, 253 (1976); *ibid.* 111, 201 (1978); *ibid.*, 123, 468 (1979).

Energy formulas of the interacting boson model

$$E^{(I)}(N,n_d,v,n_\Delta,L,M_L)=E_0+\alpha n_d(n_d+4)+\beta v(v+3)+\gamma L(L+1)$$

$$E^{(II)}(N,\lambda,\mu,K,L,M_L)=E_0+\kappa(\lambda^2+\mu^2+\lambda\mu+3\lambda+3\mu)+\kappa' L(L+1)$$

$$E^{(III)}(N,\sigma,\tau,\nu_\Delta,L,M_L)=E_0+A\sigma(\sigma+4)+B\tau(\tau+3)+CL(L+1)$$

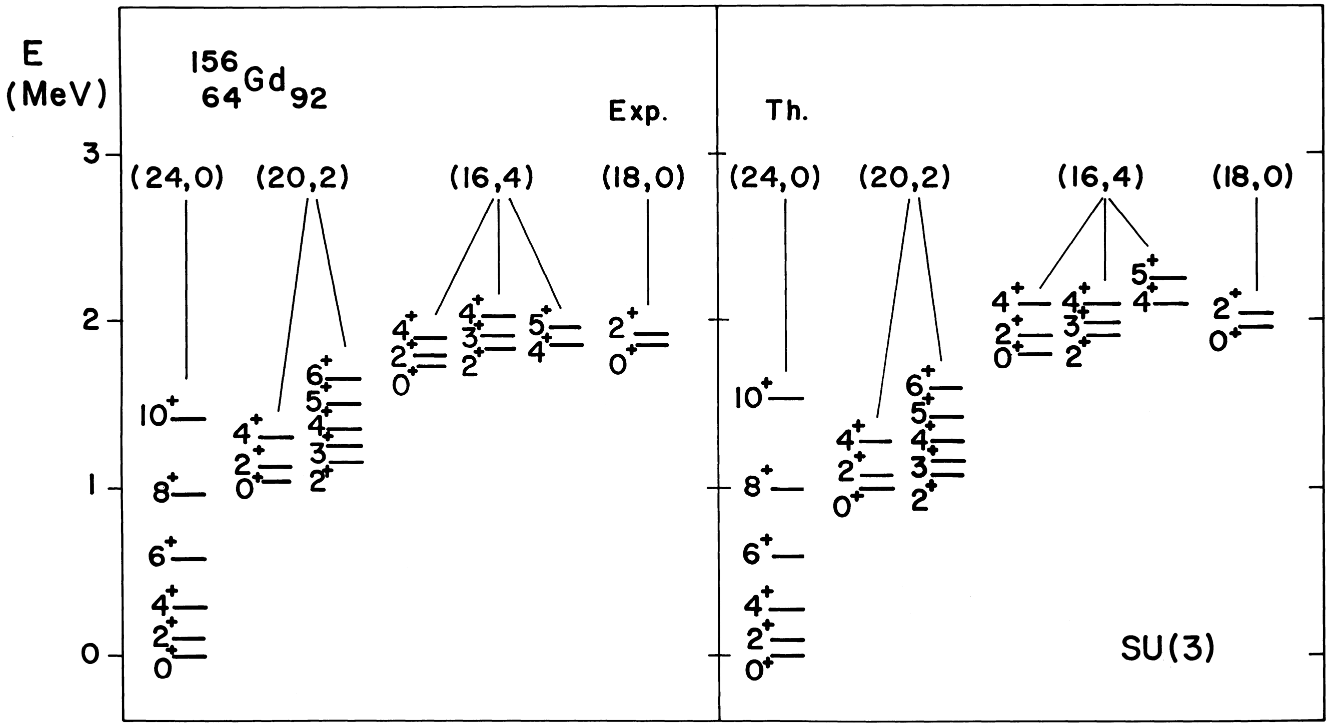


g.s. β γ ββ βγ γγ

Rotational bands with L=0,2,4,…, max{λ,μ} for K=0 and L=K, K+1,…, K+max{λ,μ} for K≠0, where K=integer=min{λ,μ}, min{λ,μ},…,1 or 0.

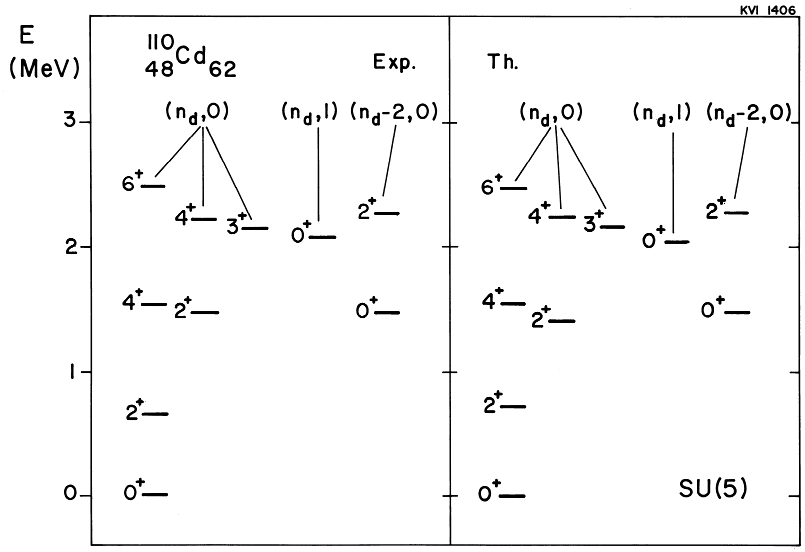
EVIDENCE FOR IBM SYMMETRIES IN NUCLEI

Many examples found (1974-…) SU(3)

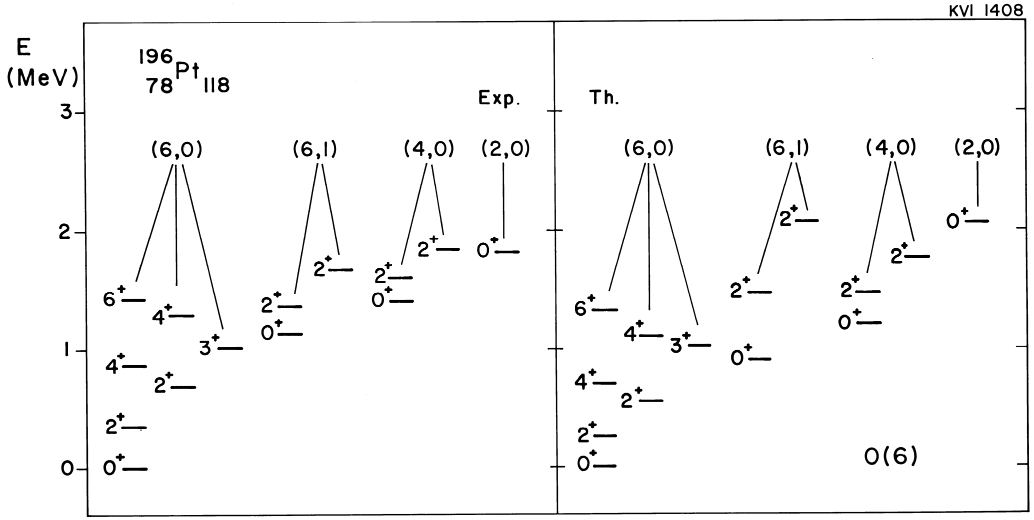


Symmetry extends to higher energies and is more accurate than originally thought!

Among the best examples of symmetry in physics!


KVI 1406
E (MeV)
$^{110}_{48}Cd_{62}$
Exp.
Th.
$(n_d,0)$
$(n_d,1)$
$(n_d-2,0)$
SU(5)

U(5)


KVI 1408
E (MeV)
$^{196}_{78}Pt_{118}$
Exp.
Th.
(6,0)
(6,1)
(4,0)
(2,0)
O(6)

SO(6)

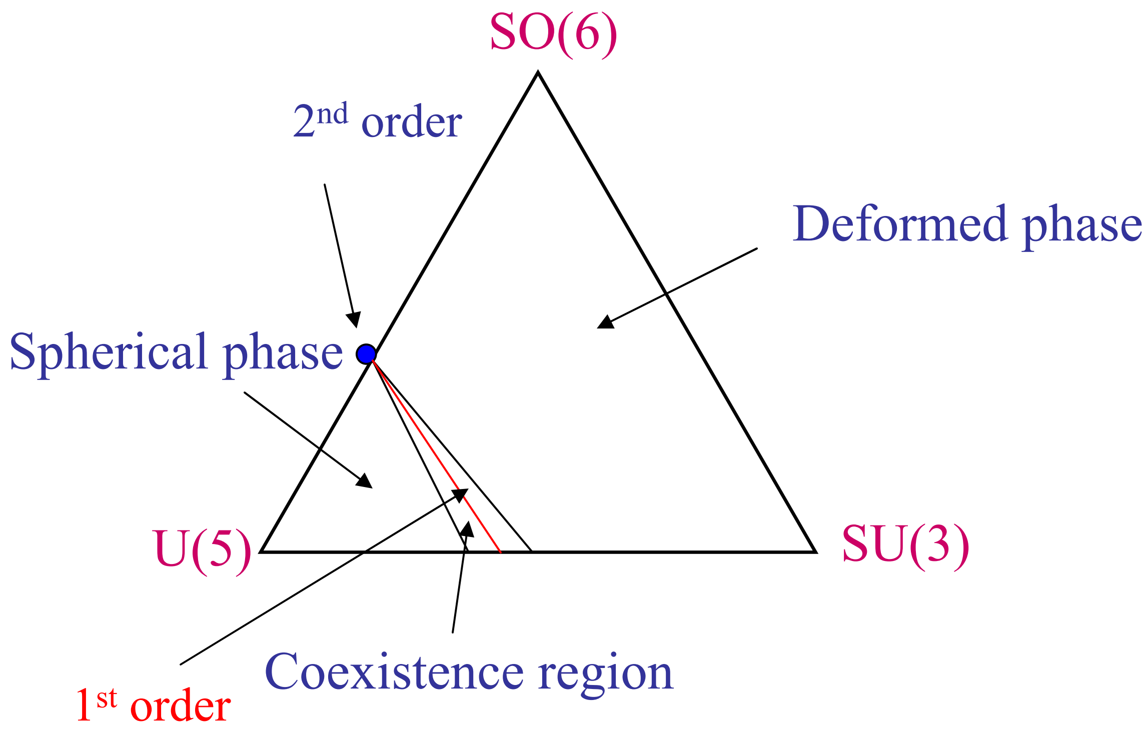
QUANTUM PHASE TRANSITIONS (QPT)

QPTs are phase transitions that occur as a function of a control parameter, ξ, in the Hamiltonian H describing the system

$$H = \left(1 - \xi\right) H_1 + \xi H_2$$

Dynamic symmetries provide also a classification of quantum phase transitions, and allow the construction of the phase diagram.

PHASE DIAGRAM OF NUCLEI IN THE INTERACTING BOSON MODEL

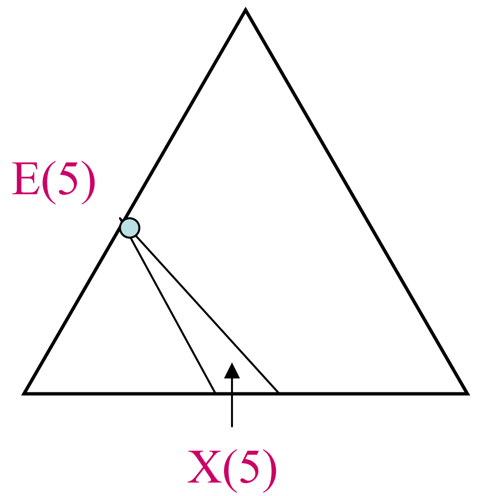


An intriguing (and surprising) result[¶] :

The structure of a system at the critical point of a quantum phase transition is simple. The energy eigenvalues are given by zeros of Bessel functions! Extends the notion of symmetry to the most difficult situation encountered in quantum physics.

⇒ Critical symmetry associated with scale invariance (Conformal invariance in quantum field theory)

CRITICAL "SYMMETRIES" OF THE INTERACTING BOSON MODEL

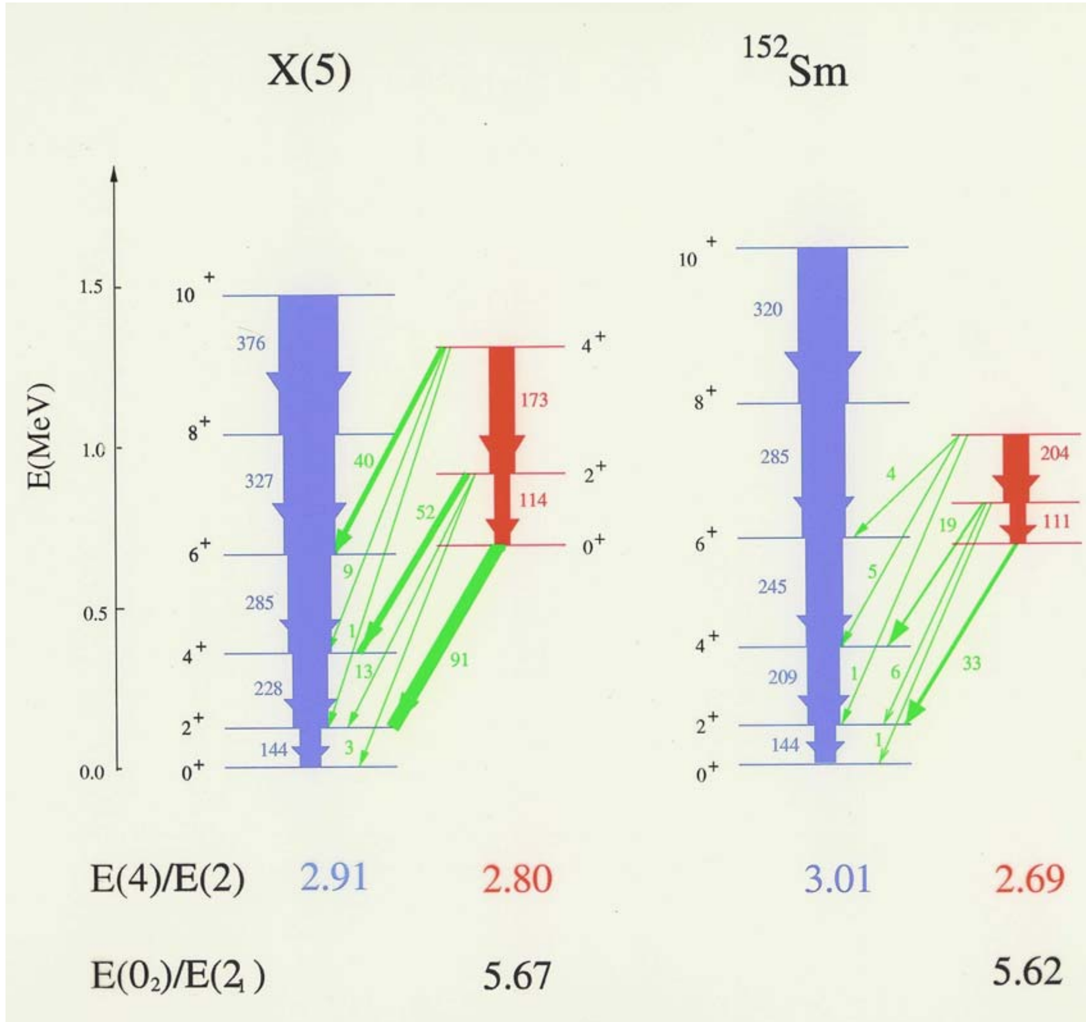


¶ F. Iachello, Phys. Rev. Lett. 85, 3580 (2000); *ibid.* 87, 052502 (2001); *ibid.* 91, 132502 (2003).

EVIDENCE FOR CRITICAL SYMMETRY IN NUCLEI

Several examples found (2000-…)

X(5)



R.F. Casten and N.V. Zamfir, Phys. Rev. Lett. 87, 052503 (2001).

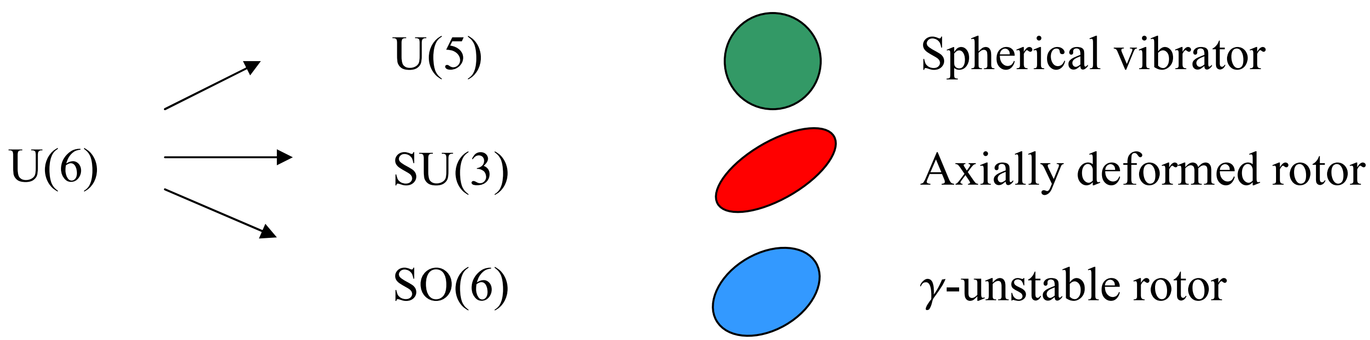
Figure courtesy of N.V. Zamfir

GEOMETRY

Associated with any algebra g, there is a geometry.
A space can be constructed through the use of
coherent or intrinsic states

$$\left|N;\alpha_\mu\right\rangle = \left(s^\dagger + \sum_\mu \alpha_\mu d_\mu^\dagger\right)^N \left|0\right\rangle$$

Shapes associated with the symmetries of the IBM



SUMMARY OF SYMMETRIES OF THE COLLECTIVE MODEL

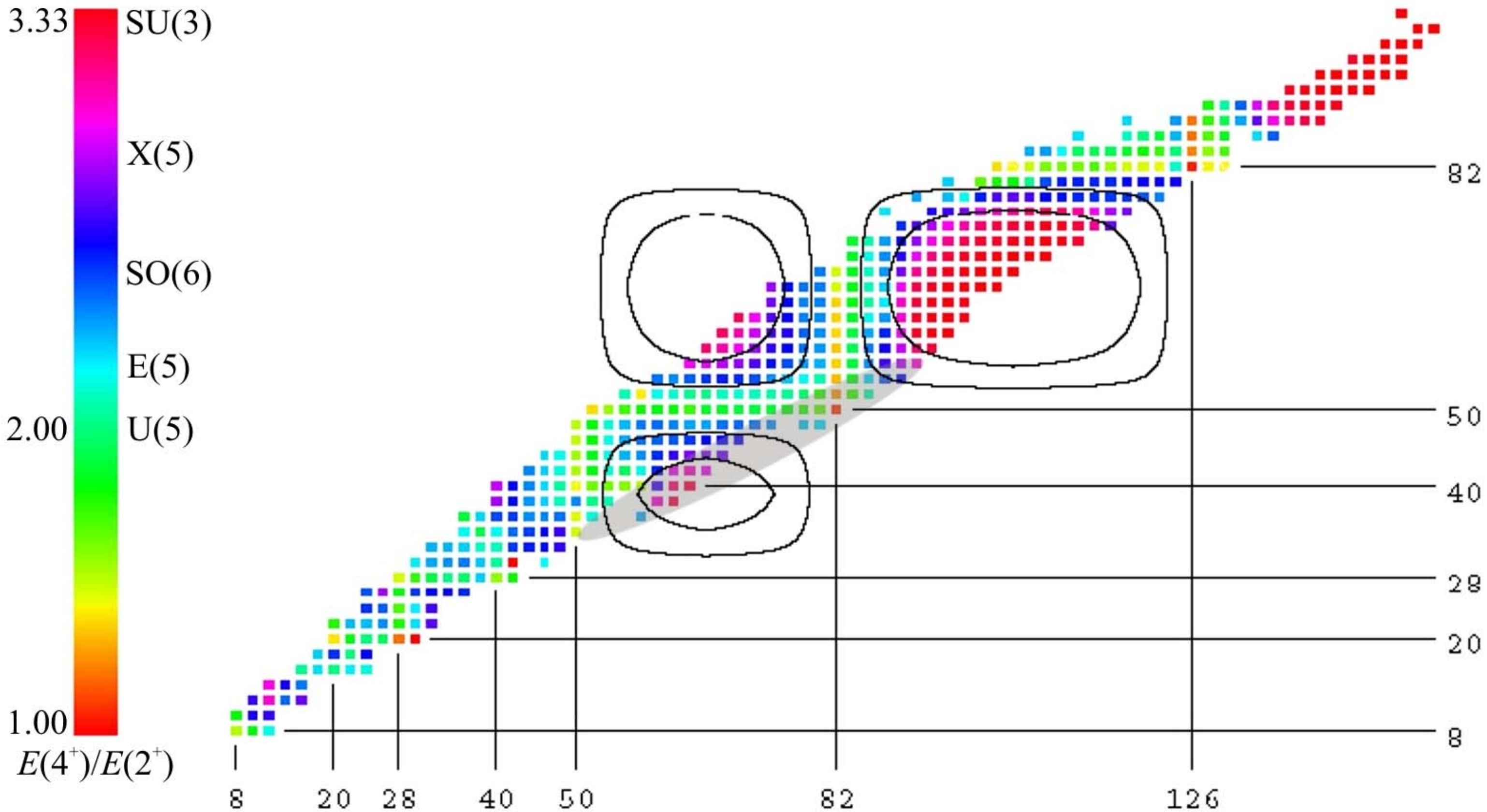


Figure courtesy of P. van Isacker

CONTINUOUS SYMMETRIES: THE INTERACTING BOSON MODEL-2

Building blocks:
Proton, π, and neutron, ν, pairs with J=0 and J=2 treated as bosons s_{π} ,$d_{\pi,\mu}$,$s_{\nu,\mu}$,$d_{\nu,\mu}$ (μ=0,$\pm$1,$\pm$2)

Algebraic structure (spectrum generating algebra)

$$U_{\pi}(6)\otimes U_{\nu}(6)$$

[The algebra is actually the direct sum of two U(6) algebras,

$$u_{\pi}(6)\oplus u_{\nu}(6)$$

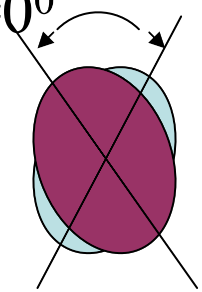
The basis states are the direct product of representations of two U(6) groups].

In systems composed of two sub-systems new symmetries occur

$$U_\pi(6) \otimes U_\nu(6) \rightarrow U_{\pi+\nu}(6) \rightarrow \begin{cases} U_{\pi+\nu}(5) \\ SU_{\pi+\nu}(3) \\ SO_{\pi+\nu}(6) \end{cases}$$

Two-fluid vibrator

$\varphi_e = 0^0$



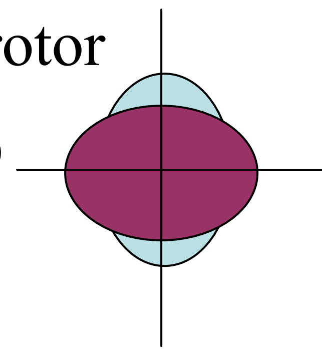
Two-fluid axial rotor

Two-fluid γ-unstable rotor

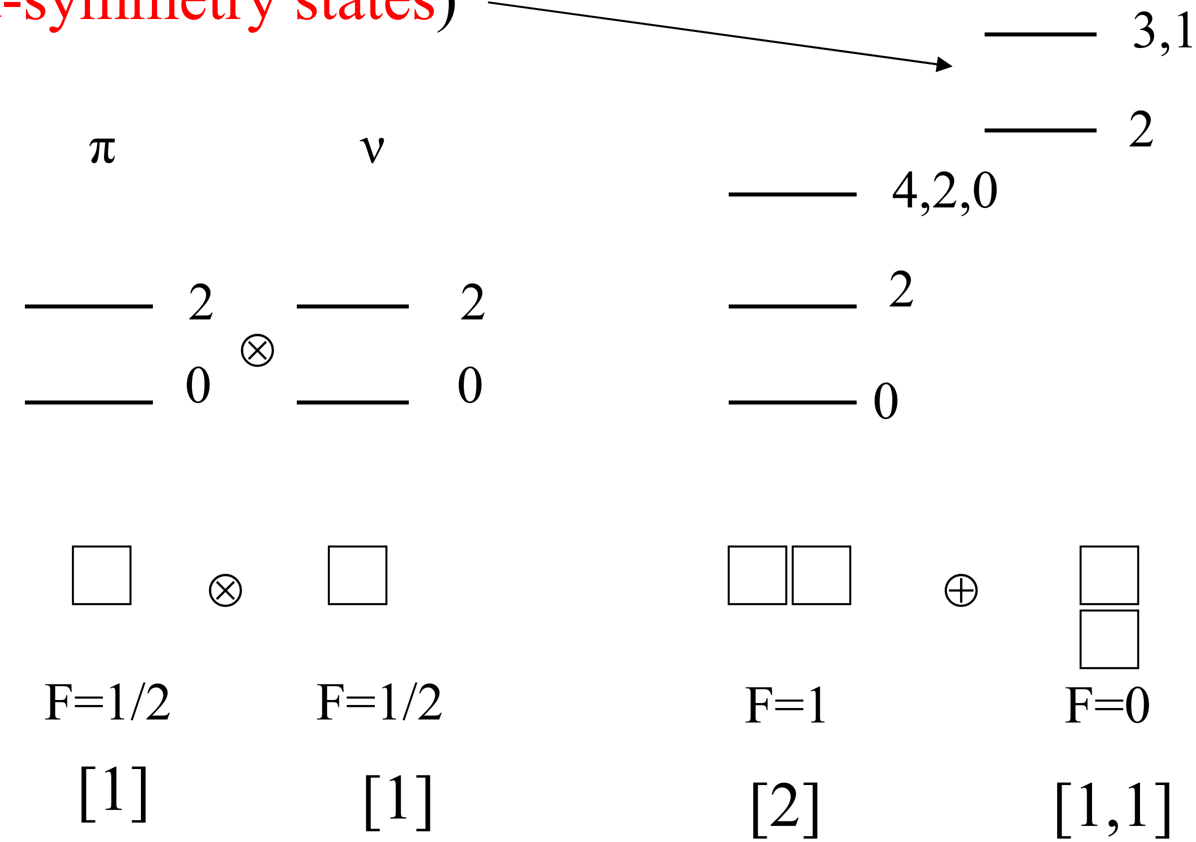
$$\begin{array}{ccc} U_\pi(6) \rightarrow SU_\pi(3) & & \\ \otimes & \searrow & \\ & & SU_{\pi+\nu}(3)^* \\ U_\nu(6) \rightarrow \overline{SU_\nu(3)} & \nearrow & \end{array}$$

Two-fluid triaxial rotor

$\varphi_e = 90^0$



The combination of the two bosonic subsystems produces states with symmetry other than totally symmetric (mixed-symmetry states)



States can be labeled either by their symmetry character (Young tableau) or by the value of the F-spin (isospin of bosons).

PROTON-NEUTRON (F-SPIN) SYMMETRY

In nuclei, composed of protons and neutrons, the combination of the two subsystems leads to a variety of new physical phenomena: mixed symmetry states, scissor modes, triaxiality.

Dynamic symmetries provide benchmarks for the study of these phenomena

DYNAMIC SYMMETRIES OF IBM-2

$U_{\pi\nu}(5)$ $SU_{\pi\nu}(3)$ $SO_{\pi\nu}(6)$

VOLUME 53, NUMBER 15 PHYSICAL REVIEW LETTERS 8 OCTOBER 1984

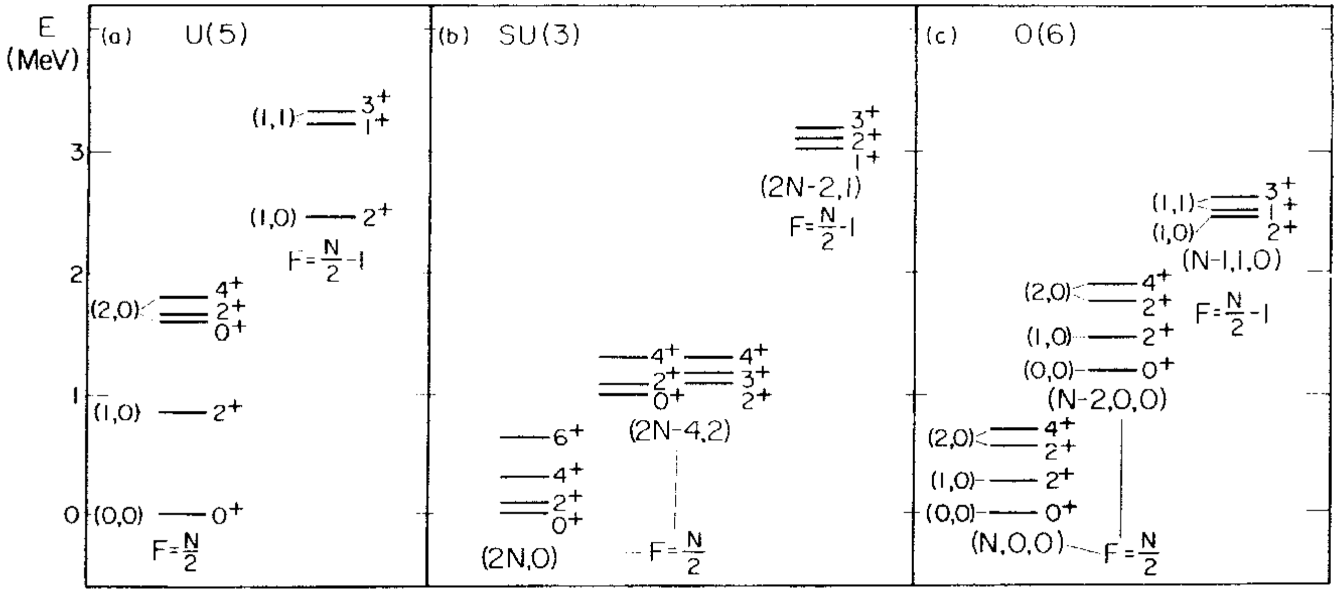
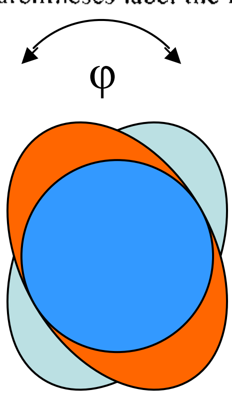


FIG. 1. Structure of F-vector excitations in (a) vibrational nuclei, (b) rotational nuclei with axial symmetry, and (c) rotational nuclei with γ instability. The numbers in parentheses label the representations of the appropriate groups and are discussed in Refs. 9, 12, and 13.



EVIDENCE FOR IBM-2 SYMMETRIES

Mixed symmetry states in deformed nuclei were discovered in 1984 by Richter *et al.*, and in spherical nuclei in 1999 by Pietralla *et al.*

$U_{\pi\nu}(5)$

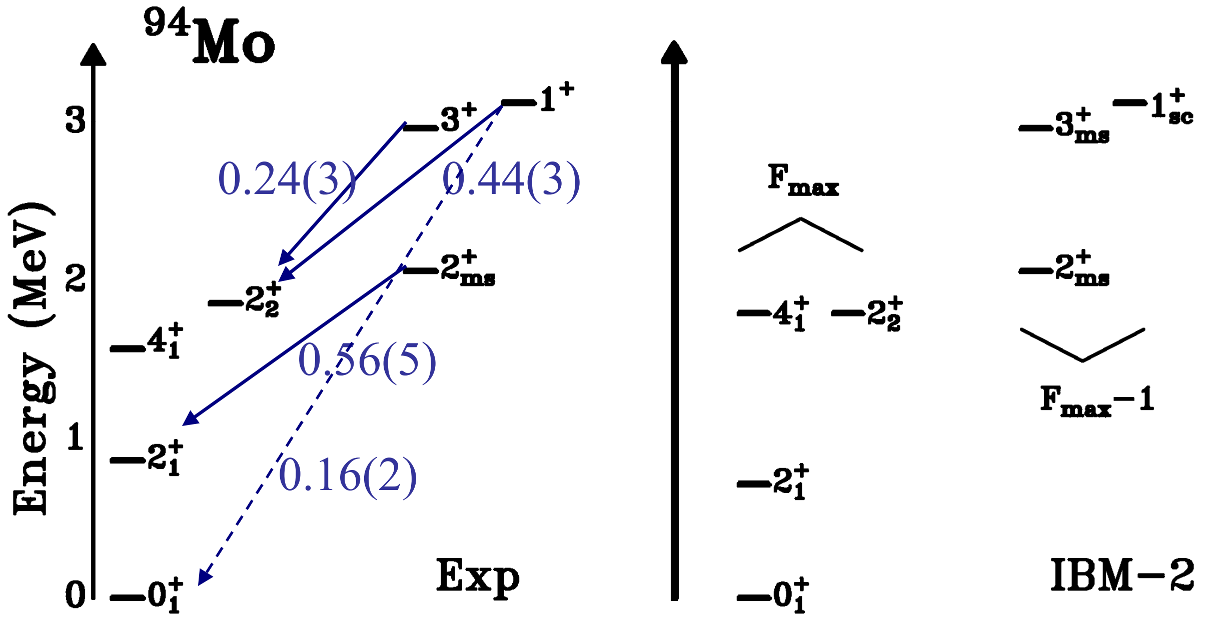


Figure from N. Pietralla, P. von Brentano and A.F. Lisetskiy, Progress in Particle and Nuclear Physics, 60, 225 (2008).

ROLE OF DISCRETE SYMMETRIES IN NUCLEAR PHYSICS

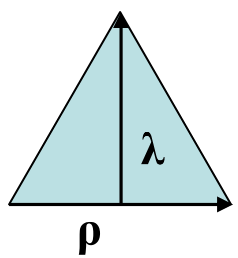
Early work:

1937 Wheeler
1954 Dennison

DISCRETE SYMMETRIES OF THE ALGEBRAIC CLUSTER MODEL

Discrete symmetries of the (algebraic) cluster model (ACM)¶ [α-particle model of light nuclei] in terms of representations of U(3k-2): bosonic quantization of the Jacobi variables

k	Nucleus	U(3k-2)	Discrete symmetry	Jacobi variables
2 α	^{8}Be	U(4)#	Z_2	$\boldsymbol{\rho}$
3 α	^{12}C	U(7)¶	D_3	$\boldsymbol{\rho},\boldsymbol{\lambda}$
4 α	^{16}O	U(10)§	T_d	$\boldsymbol{\rho},\boldsymbol{\lambda},\boldsymbol{\eta}$



F. Iachello, Chem. Phys. Lett. 78, 581 (1981); Phys. Rev. C23, 2778 (1981).

¶ R. Bijker and F. Iachello, Phys. Rev. C61, 067305 (2000).
R. Bijker and F. Iachello, Ann. Phys. (N.Y.) 298, 334 (2002).

§ R. Bijker and F. Iachello, Phys. Rev. Lett. 112, 152501 (2014).

Method for constructing representations of a discrete group G

Diagonalization of the symmetry adapter operators

For the cluster model with identical constituents (α-particles) it is sufficient to diagonalize the symmetry adapter operators of the permutation group S_n.
For the ACM, formulated in terms of U(3k-2) the construction is even simpler, since U(3k-2) contains the harmonic oscillator group U(3k-3) and the breaking of U(3k-3) onto S_n was studied years ago by Kramer and Moshinsky[¶] .
One can therefore find the angular momentum and parity, L^P, content in a given representation of the discrete group G.

Group G	Symmetry adapter
$Z_2 \sim S_2$	Transposition (12)
$D_3 \sim S_3$	Transposition (12), Cyclic permutation (123)
$T_d \sim S_4$	Transposition (12), Cyclic permutation (1234)

¶ P. Kramer and M. Moshinsky, Nucl. Phys. 82, 241 (1966).

Representations can be labeled either by S_n or by the isomorphic discrete group G

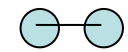
Dictionary:

Group G	G label	S_n label	Degeneragy
$Z_2 \sim S_2 \sim P$	A	[2]	singly degenerate
$D_3 \sim S_3$	A	[3]	singly degenerate
	E	[21]	doubly degenerate
$T_d \sim S_4$	A	[4]	singly degenerate
	F	[31]	triply degenerate
	E	[22]	doubly degenerate

Young tableaux

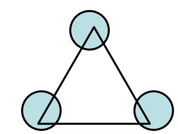
Energy formulas ("dynamic symmetries") for a *rigid* roto-vibrator composed of kα-particles:

2 α $\rightarrow$ $$E(v,L) = E_0 + \omega\left(v + \frac{1}{2}\right) + \kappa L(L+1)$$



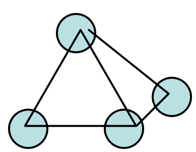
Z_2

3 α $\rightarrow$ $$E(v_1,v_2,L) = E_0 + \omega_1\left(v_1 + \frac{1}{2}\right) + \omega_2\left(v_2+1\right) + \kappa L(L+1)$$



D_{3h}

4 α $\rightarrow$ $$E(v_1,v_2,v_3,L) = E_0 + \omega_1\left(v_1 + \frac{1}{2}\right) + \omega_2\left(v_2+1\right) + \omega_3\left(v_3 + \frac{3}{2}\right) + \kappa L(L+1)$$

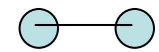


T_d

It is also possible to calculate all properties, B(EL), form factors in electron scattering, etc., in explicit analytic form.

2 α: ^{8}Be

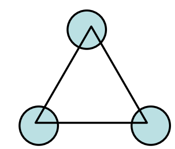
$$B(EL;0\rightarrow L)=\left(\frac{Ze\beta^{L}}{2}\right)^{2}\frac{(2L+1)}{4\pi}\left[2+2P_{L}(-1)\right]$$



Z_2

3 α : ^{12}C

$$B(EL;0\rightarrow L)=\left(\frac{Ze\beta^{L}}{3}\right)^{2}\frac{(2L+1)}{4\pi}\left[3+6P_{L}\left(-\frac{1}{2}\right)\right]$$



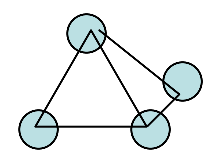
D_{3h}

$$\boxed{F_{L}(0^{+}\rightarrow L^{P};q)=c_{L}\,j_{L}(q\beta)}$$

$$\boxed{c_{0}^{2}=1,c_{2}^{2}=\frac{5}{4},c_{3}^{2}=\frac{35}{8},c_{4}^{2}=\frac{81}{64}}$$

4 α : ^{16}O

$$B(EL;0\rightarrow L)=\left(\frac{Ze\beta^{L}}{4}\right)^{2}\frac{(2L+1)}{4\pi}\left[4+12P_{L}\left(-\frac{1}{3}\right)\right]$$

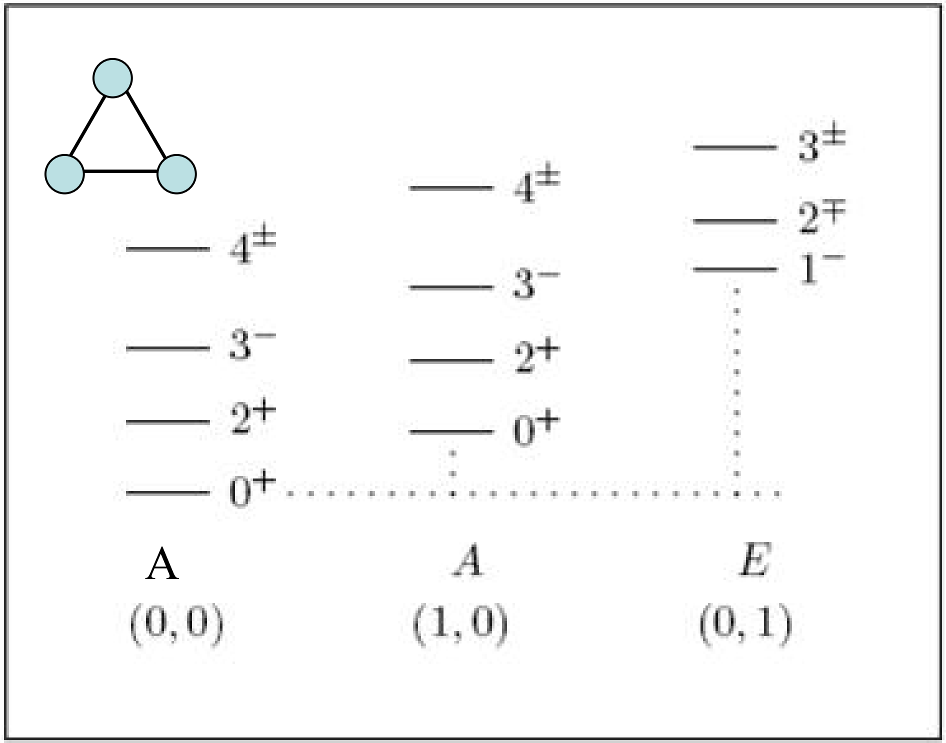


T_d

$$\boxed{F_{L}(0^{+}\rightarrow L^{P};q)=c_{L}\,j_{L}(q\beta)}$$

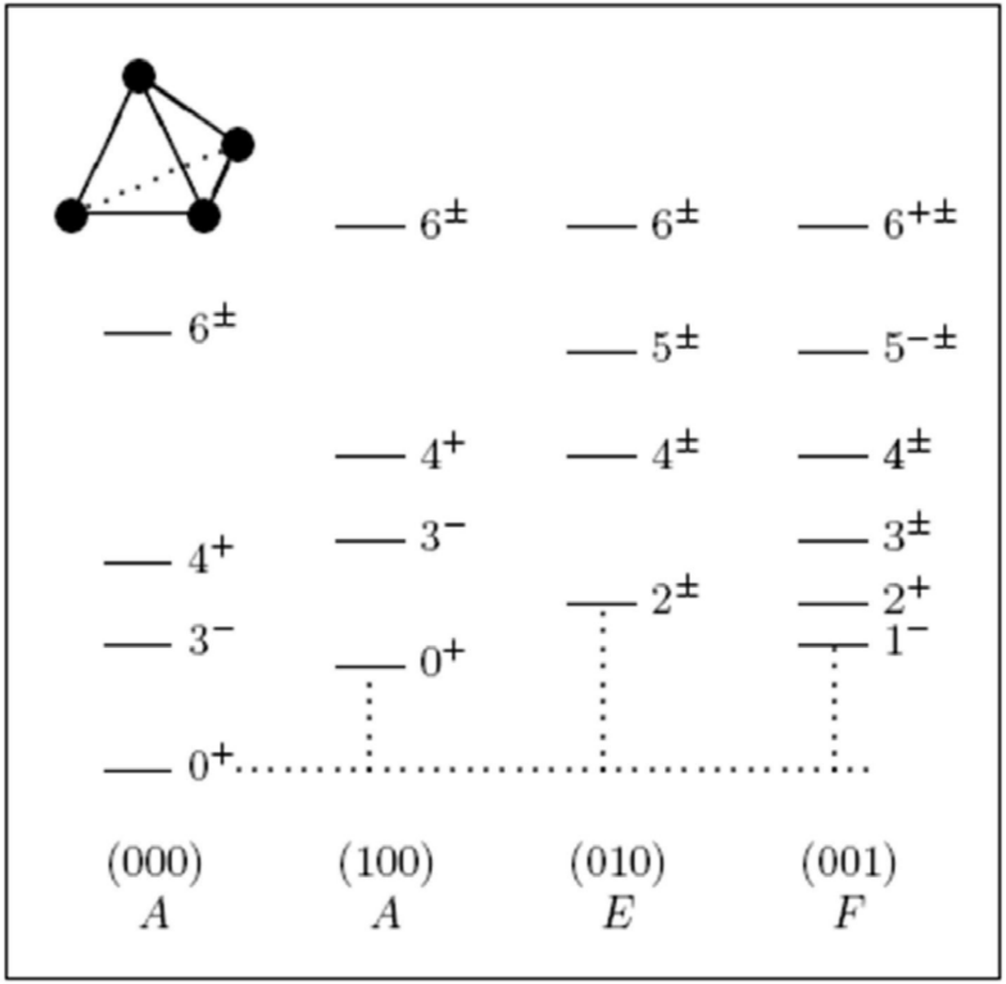
$$\boxed{c_{0}^{2}=1,c_{3}^{2}=\frac{35}{9},c_{4}^{2}=\frac{7}{3},c_{6}^{2}=\frac{32}{81}}$$

^{12}C



A irreps: Rotational bands with $L^P = 0^+ , 2^+ , 3^- , 4^\pm$, … (K=0,3,6,…)
E irreps: 1^-, $2^\pm$, $3^\pm$, … (K=1,2,4,5, …)
Both positive and negative parity states sit on the same rotational band because of the lack of reflection symmetry of a D_3 configuration!
Rotational bands have parity doubling!

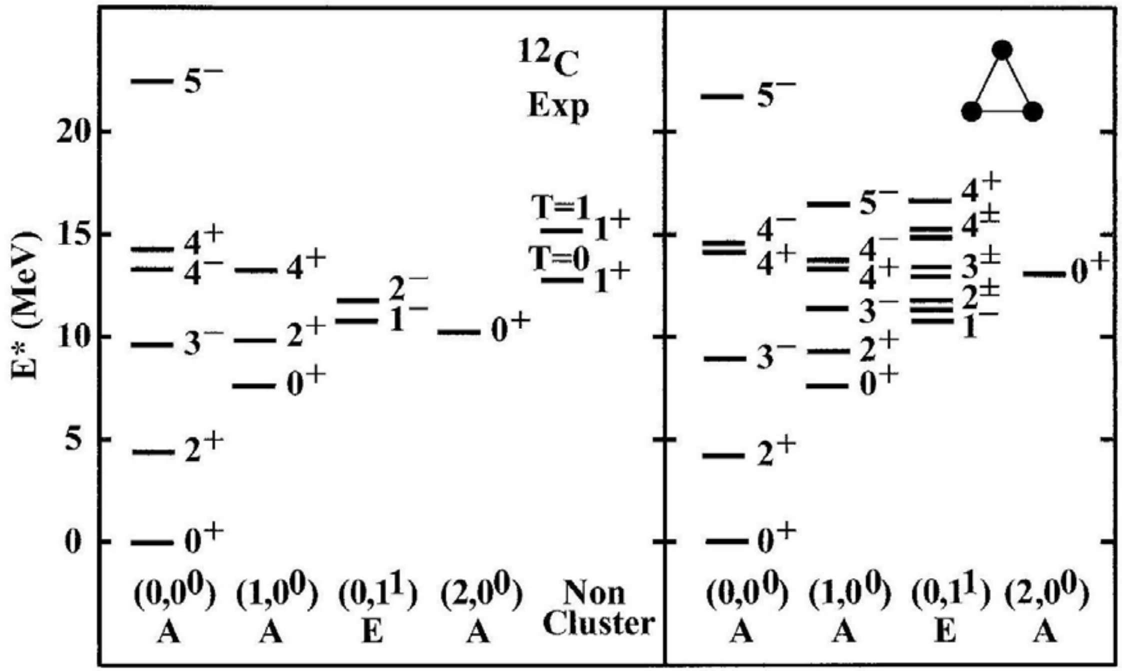
^{16}O



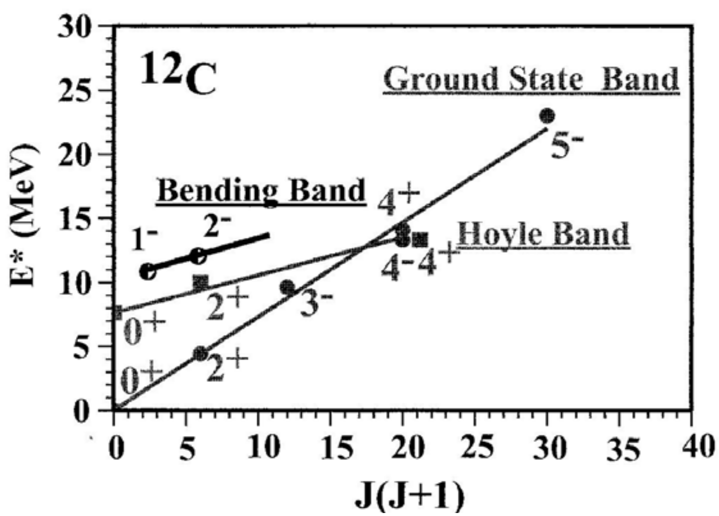
A irreps: Rotational band with 0^+, 3^- , 4^+ , $6^\pm$, …
E irreps: $2^\pm$, $4^\pm$, $5^\pm$, $6^\pm$, … ; F irreps: 1^-, 2^+, $3^\pm$, $4^\pm$, …
Both positive and negative parity states on the same rotational band!
Parity doubling!

EVIDENCE FOR D_3 SYMMETRY

^{12}C



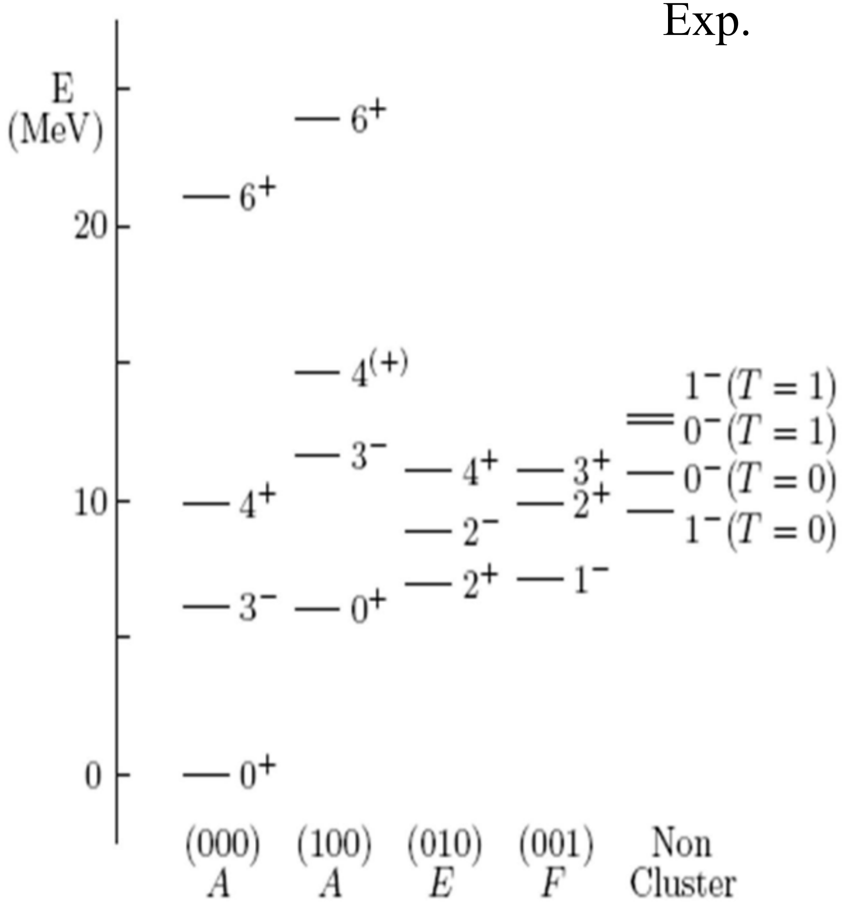
Observed cluster rotational bands →



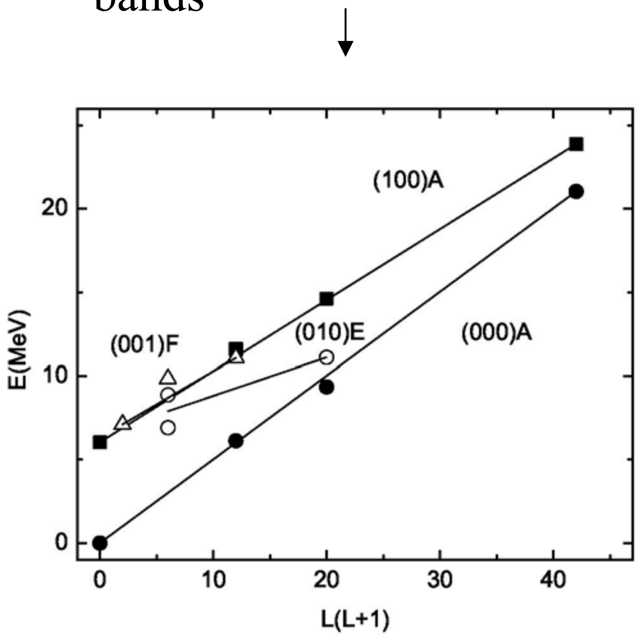
From D.J. Marin-Lambarri *et al.*, Phys. Rev. Lett. 113, 012502 (2014).

^{16}O EVIDENCE FOR T_d SYMMETRY

Exp.



Observed cluster rotational bands



From R. Bijker and F. Iachello, Phys. Rev. Lett. 112, 152501(2014).

The occurrence of clustering with discrete symmetry is confirmed by the B(EL) values along the ground state band

^{16}O

B(EL; $L^P \to 0^+$)	Th	Exp	E(L^P)	Th*	Exp
B(E3; $3^- \to 0^+$)	181	205±10	E(3^-)	6132	6130
B(E4:$4^+ \to 0^+$)	338	378±133	E(4^+)	10220	10356
B(E6:$6^+ \to 0^+$)	8245		E(6^+)	21462	21052

Parameter free: consequence of symmetry alone!

B(EL) values in e^2fm^{2L} and E in keV

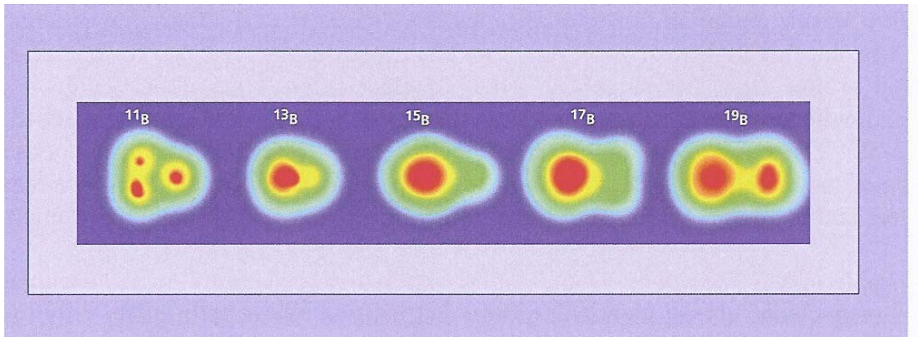
β=2.0fm extracted from the elastic form factor measured in electron scattering

* E(keV)=511 L(L+1)

The occurrence of clustering is also confirmed by microscopic calculations:

(i) Molecular dynamics method (Feldmeyer *et al.*)¶

This method interprets the structure of all light nuclei (not only n α) in terms of clusters.



Boron clusters

It confirms D_3 symmetry in the g.s. of ^{12}C !

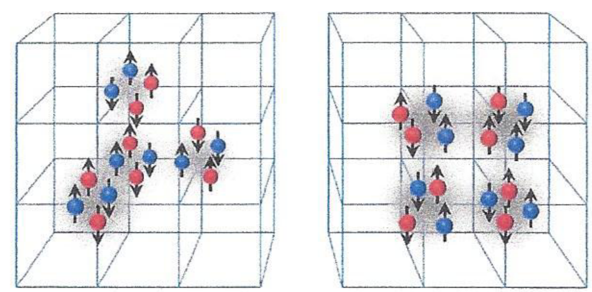
Very recently by:

16 Oxygen clusters

(ii) Lattice EFT method (Epelbaum *et al.*)§

This method confirms T_d symmetry in the g.s. of ^{16}O !

[However, lattice calculations cannot address states with J>2, nor parity doubled states, which are the spectroscopic signatures of clustering.]



(a) Initial state "A", 8 equivalent orientations.

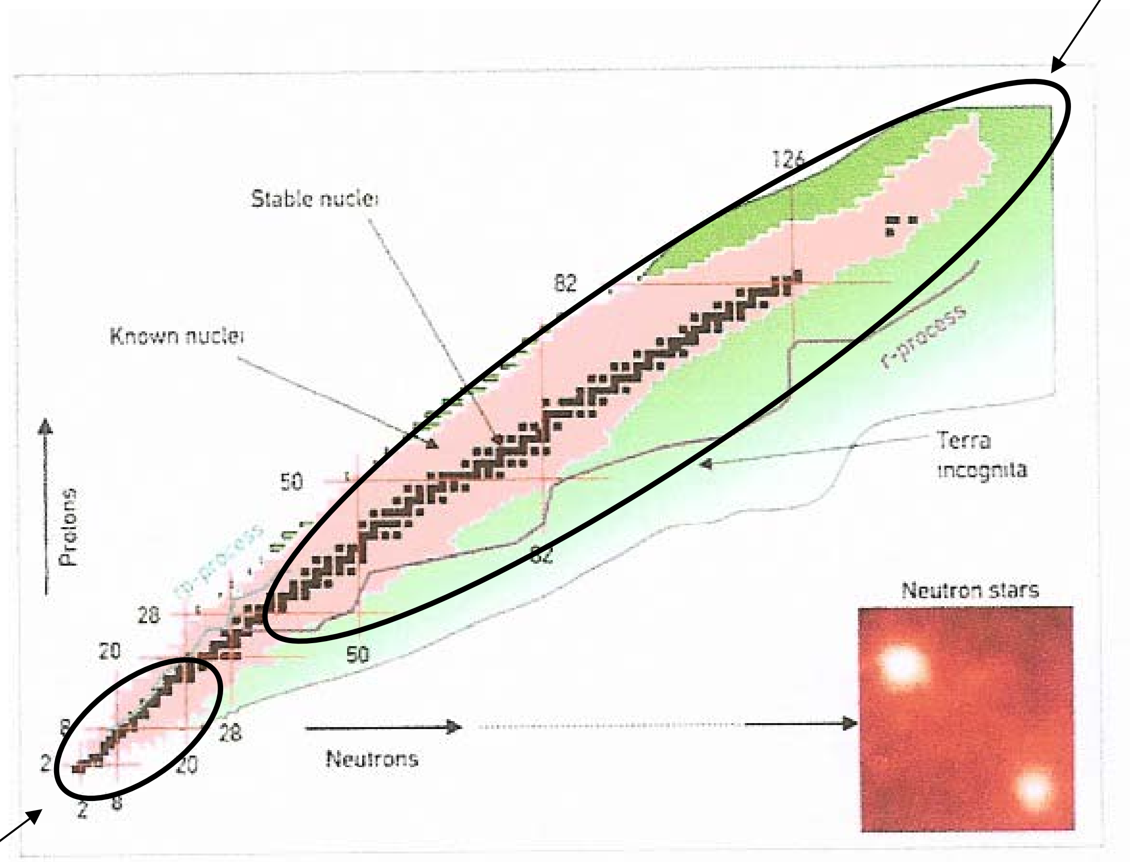
(b) Initial states "B" and "C", 3 equivalent orientations.

¶ H. Feldmeyr and T. Neff, Proc. of the International School "Enrico Fermi", Course CLXIX, IOS Press, Amsterdam, pp. 185-215.

§ E. Epelbaum *et al.*, Phys. Rev. Lett. 112, 102501 (2014).

SUMMARY OF THE ROLE OF SYMMETRY IN NUCLEAR PHYSICS

Dominated by symmetries of the collective model (IBM)



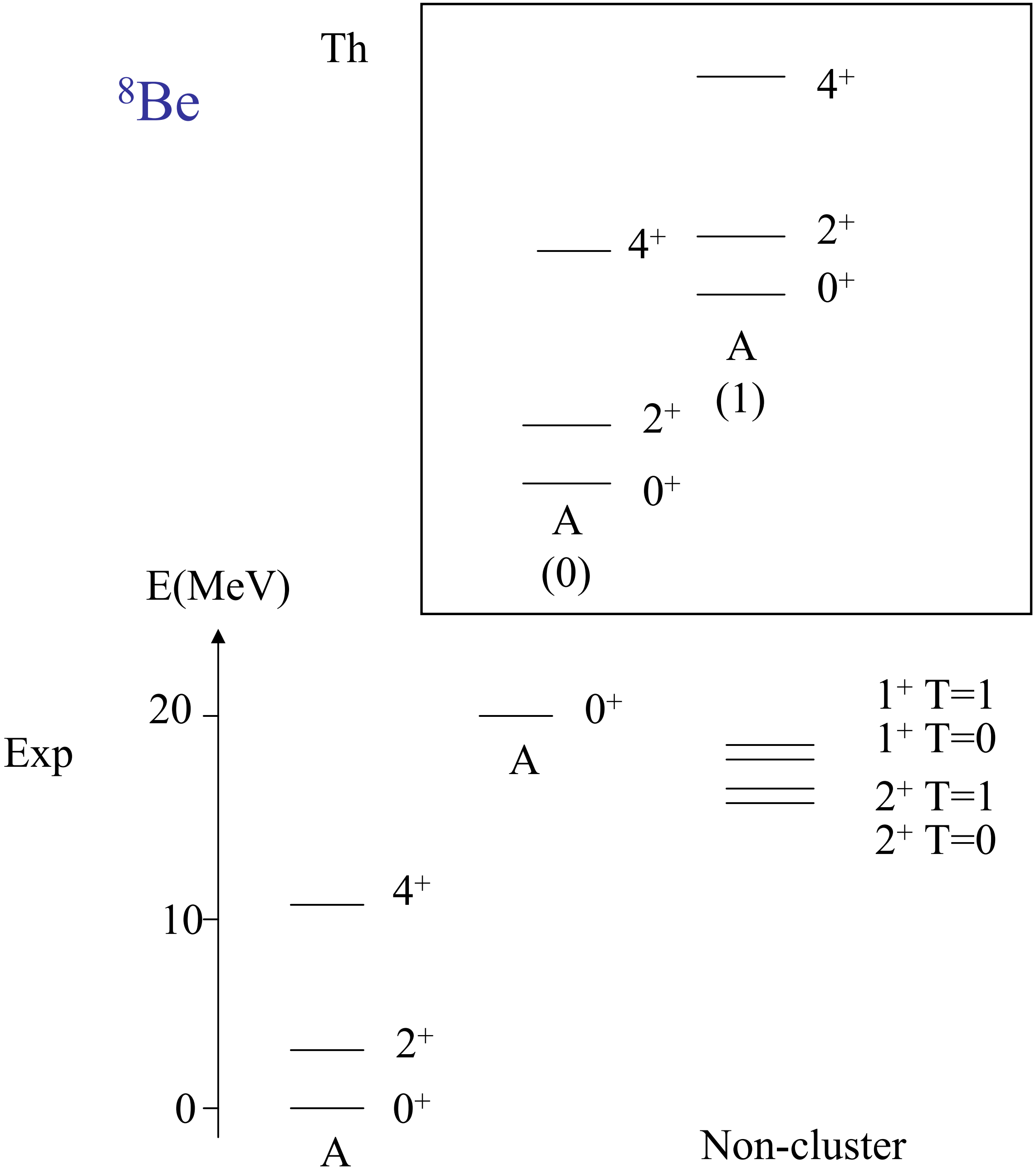
Dominated by symmetries of the cluster model (ACM)

CONCLUSIONS

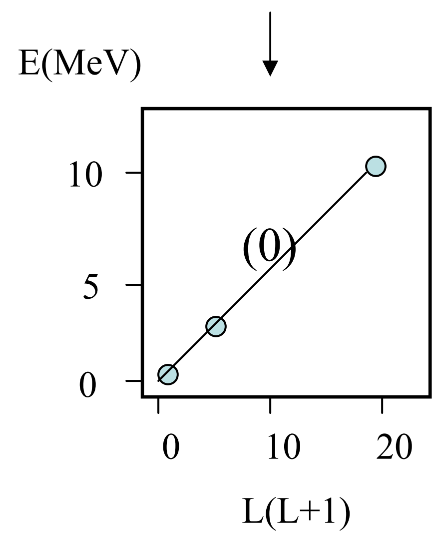
Symmetries (both continuous and discrete) are pervasive in nuclear physics.
They are much more accurate and extend to much higher energies than originally thought.
Simplicity in complexity

Symmetries provide benchmarks for the analysis of experimental data and give clues for a microscopic understanding of the structure of physical systems.

^{8}Be



Observed cluster rotational bands



MICROSCOPIC DESCRIPTION OF CLUSTERING

A shell model description of cluster states is very difficult!

^{12}C $\quad 0^+$ 7.654 $\rightarrow$ 4p-4h
$\quad 5^-$ 22.4 $\rightarrow$ 5p-5h

^{16}O $\quad 0^+$ 6.049 $\rightarrow$ 4p-4h ¶
$\quad 1^-$ 7.116 $\rightarrow$ 5p-5h

¶ G.E. Brown and A.M. Green, Nucl. Phys. 75, 401 (1966).
H. Feshbach and F. Iachello, Phys. Lett. B45, 7 (1973).

A challenge for large scale shell model and no-core shell model!

NON-CLUSTER STATES

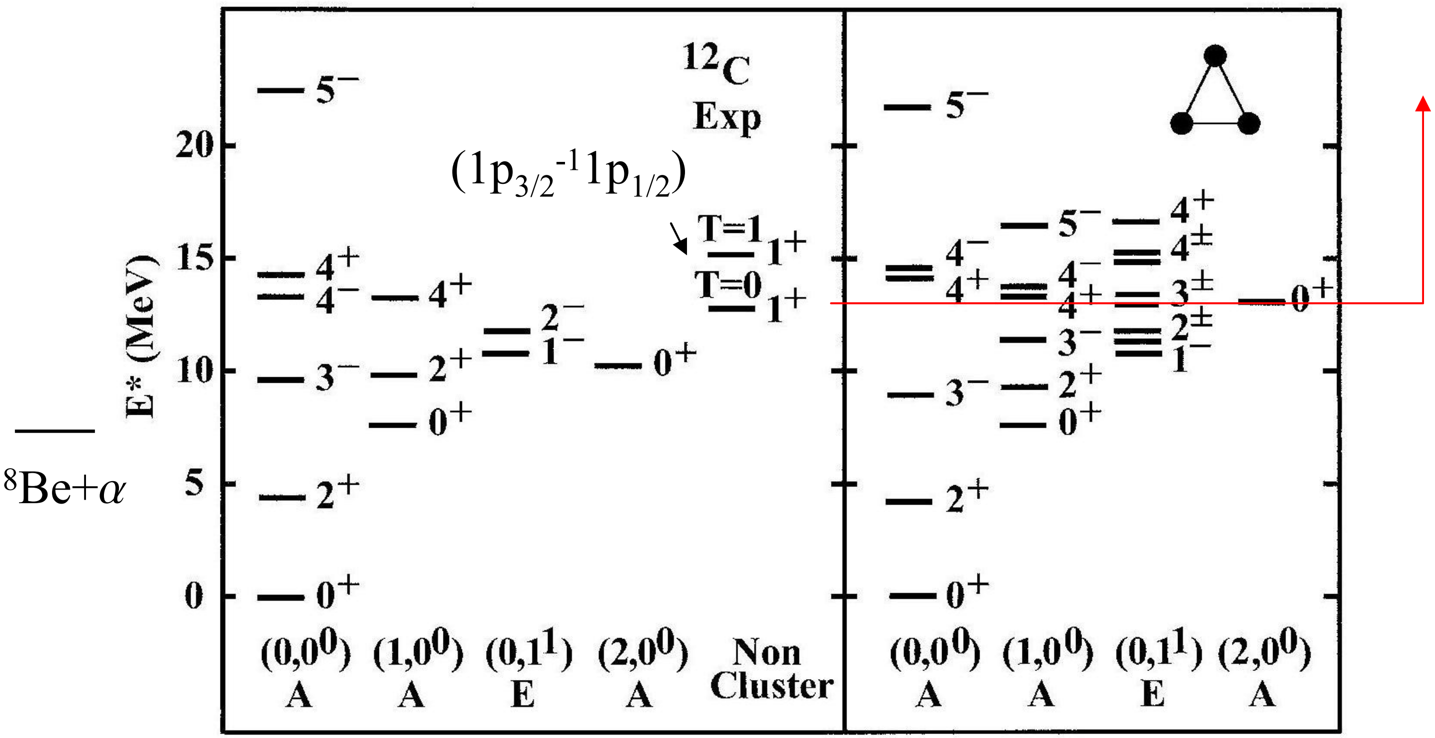
In addition to cluster states there are non-cluster states. Non-cluster states can be in some cases clearly identified, since some states are forbidden by the discrete symmetry.

For ^{12}C, 1^{+} states cannot be formed in D_3.
For ^{16}O, 0^{-} states cannot be formed in T_d.

These are signatures of non-cluster states.

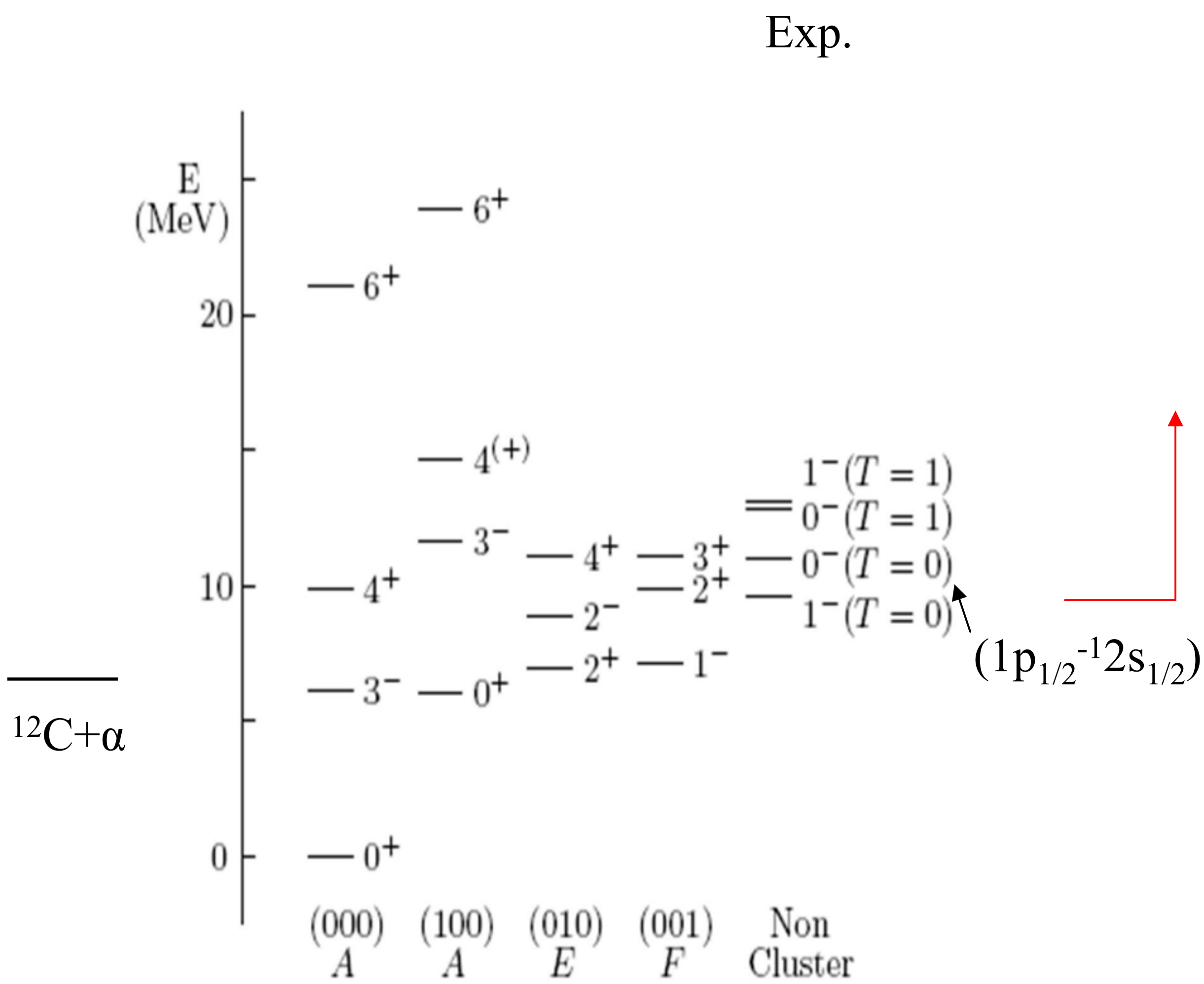
Also, with constituent α-particles we cannot form T=1 states, another signature of non-cluster states.

^{12}C



Above this energy cluster and non-cluster states coexist

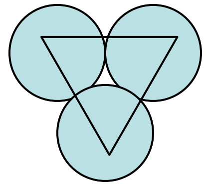
^{16}O



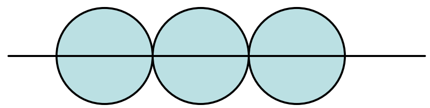
Above this energy, cluster and non-cluster states coexist

Also the structure of cluster states above the threshold for α-emission can be greatly modified from the simple rigid structure and can have a different geometric configuration.

^{12}C

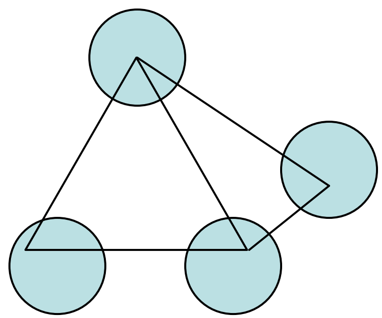


D_3

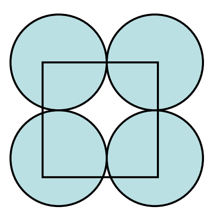


$C_{\infty h}$

^{16}O



T_d



D_4