Invited Talks

Orthogonal Polynomials and Quadrature

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Abstract

Various concepts of orthogonality on the real line are reviewed that arise in connection with quadrature rules. Orthogonality relative to a positive measure and Gauss-type quadrature rules are classical. More recent types of orthogonality include orthogonality relative to a sign-variable measure of interest in connection with Gauss-Kronrod quadrature, and power orthogonality for Turán-type quadrature. Relevant questions of numerical computation are also considered.

Gauss Quadrature for Rational Functions

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Abstract

Gauss-type quadrature rules are studied that are exact for a mixture of polynomials and rational functions, the latter being selected so as to simulate poles that may be present in the integrand. The underlying theory is presented as well as methods for constructing such rational Gauss formulae. Applications are given to the computation of generalized Fermi-Dirac and Bose-Einstein integrals.

Bounds for the Entries of Matrix Functions with Applications to Preconditioning

Gene Golub

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Abstract

Let A be a symmetric matrix and let f be a smooth function defined on an interval containing the spectrum of A. The entries of the matrix $f(A)$ can be expressed as Riemann–Stieltjes integrals of f with respect to a suitable measure. By approximating these integrals with Gauss–type quadrature rules one obtains bounds or estimates for the entries of $f(A)$. These quadrature rules can be evaluated by means of the Lanczos process. Explicit bounds and estimates are obtained after a single Lanczos step for a wide class of functions f. More refined approximations or tighter bounds can be obtained by taking a few Lanczos steps. In this talk, a few choices of f of interest for preconditioning are considered. A new result on the decay of entries of analytic functions of band matrices is proved, which justifies in many cases the use of banded approximations to $f(A)$. The effectiveness of this approach will be illustrated by numerical examples. This work is in collaboration with Michele Benzi (Los Alamos National Laboratory).

Inverting Shape From Moments

Gene Golub

Computer Science Dept, Stanford University, Stanford, USA.

Abstract

Computation of certain numerical quadratures on polygonal regions of the plane and the reconstruction of these region from their moments can be viewed as dual problems.

In this talk, we discuss this idea and address the inverse problem of reconstructing a region in the complex plane from a finite number of its complex moments. The numerical computations involved in the algorithms can be very ill-conditioned. We have managed to derive inversion algorithms, based on matrix pencils, with improved stability, and have come to recognize when the problem will be ill-conditioned. We will also briefly discuss an application to a geophysical inversion problem. (Joint work with Peyman Milanfar and Jim Varah)

Software for the Algorithmic Work with Orthogonal Polynomials and Special Functions I and II

Prof. Dr. Wolfram Koepf

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Abstract

In the last decade major steps towards an algorithmic treatment of orthogonal polynomials and special functions (OP & SF) have been made, notably Zeilberger's brilliant extension of Gosper's algorithm on algorithmic definite hypergeometric summation.

By implementations of these and other algorithms symbolic computation has the potential to change the daily work of everybody who uses orthogonal polynomials or special functions in research or applications. It can be expected that symbolic computation will also play an important role in on-line versions of major revisions of existing formula books in the area of OP & SF.

It this couple of talks I will present software in Maple, Mathematica and Reduce of those algorithmic techniques, in particular of Gosper's, Zeilberger's, and Petkovšek's algorithms and their q-analogoues. Some implementational details are discussed. The main emphasis, however, is given to on-line demonstrations of these algorithms using our Maple implementations (jointly with Harald Böing) covering many examples from the field of OP & SF.

The use of CAOP, a World Wide Web version of the Askey-Wilson scheme developed by René Swarttouw, is presented as well, and it's implementation is discussed.

The Basic spectral element and mortar elements methods for elliptic problems

Yvon Maday

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Abstract

The spectral element method for elliptic problem is a high order approximation method derived from the variational formulation of the problem. It combines the domain decomposition techniques through bricks (may be curved but nevertheless regular) with the high order of precision of the polynomial approximation to define an efficient way for the numerical simulation of the phenomenon.

The numerical analysis of this type of method relies heavily on basic properties of the Legendre and more generally the Jacoby family of orthogonal polynomials.

We shall present in this part the main ingredient that make understand why spectral methods work so well.

The basic spectral element method is limited to domain decompositions that satisfy the general statement of finite element type that requires that the intersection of two subdomains is either empty or a common vertex or a common edge. The mortar element method allows for more flexibility in the decomposition and allows even for coupling spectral and finite element methods. The basics of this mortar element method will be presented in the spectral framework.

The spectral element methods for resolution of the Stokes and Navier-Stokes problems

Yvon Maday

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Abstract

The extension of the spectral element method from the Laplace equation to the Stokes problem in velocity pressure formulation involves a compatibility condition between the discrete pressure space and the discrete velocity space. This condition is known as the (L.B.B) inf-sup condition. In the spectral context, we cannot avoid the analysis of this compatibility condition and the technique required for the evaluation of its dependancy in the degree of the polynomial approximation involves a lot of nice pieces of analysis. We shall sketch the basics of the theory that is currently available with a particular emphasis to the most fundamental properties.

The extension of the method to the resolution of the Navier-Stokes problem will be finally developped so as the current algorithms of resolution. Numerical results will finally illustrate the previous analysis.

Linear Operators and Compatible Polynomial Bases

Marko Petkovšek

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Abstract

In the first part of my talk, I will review the algorithms for finding "nice" explicit solutions of linear recurrences with polynomial coefficients. These include solutions which are polynomials, rational functions, hypergeometric or q-hypergeometric terms, interlacings of hypergeometric terms, and m-sparse solutions.

In the second part, I will look at power series and, more generally, polynomial series which solve linear operator equations. An operator L and a basis $\mathcal{B}$ for the polynomial space are *compatible* if the corresponding recurrence for the coefficients of a series y in the kernel of L is of finite order. Satisfactory compatibility properties are exhibited not only by the powers and the falling powers, but also by certain families of orthogonal polynomials, such as the Gegenbauer polynomials. Combined with the algorithms mentioned in the first part, this gives the possibility of finding series solutions which have "nice" coefficients with respect to a selected polynomial basis.

The Unreasonable Power of Orthogonal Polynomials in Combinatorics I and II

Doron Zeilberger

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Abstract

Great Mathematics is destined to be eventually used, often in quite a different place from where it was first created. A case in point are the q-analogues of the classical orthogonal polynomials, preached, and largely developed, by George Andrews, Dick Askey, and their students, that lead to the brilliant solution of the Refined Alternating Sign Matrix Conjecture.