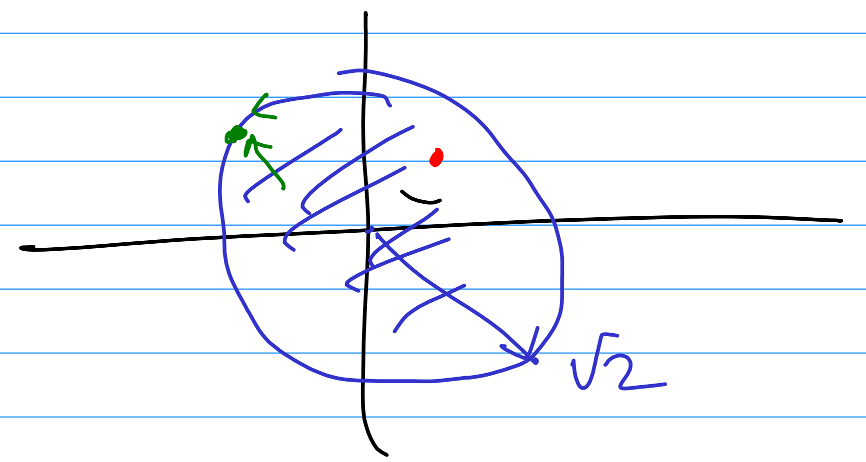
$$f(x,y) := y^2 - y + x^2 - x + 1 \qquad S = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \le 2\}$$

I) $\dfrac{\partial f}{\partial x} = 2x - 1 = 0 \qquad x = 1/2 \qquad A(1/2, 1/2)$

$\dfrac{\partial f}{\partial y} = 2y - 1 = 0 \qquad y = 1/2$



$H_f = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \qquad d^2L > 0$ en $\underline{A}$ mínimo local.

$\Downarrow$

global!!

$f = (x - 1/2)^2 + (y - 1/2)^2 + \dfrac{1}{2} \geq 1/2$

II. f pero $\phi = x^2 + y^2 - 2 = 0$

$L = x^2 - x + y^2 - y + 1 + \lambda(x^2 + y^2 - 2)$

$$\begin{cases} \dfrac{\partial L}{\partial x} = 2x - 1 + 2\lambda x = 0 & (1) \\ \dfrac{\partial L}{\partial y} = 2y - 1 + 2\lambda y = 0 & (2) \\ \dfrac{\partial L}{\partial \lambda} = x^2 + y^2 - 2 = 0 & (3) \end{cases}$$

$(1)\quad 2x(\lambda+1)=1 \qquad x=\dfrac{1}{2(\lambda+1)} \qquad (2)\Rightarrow \quad y=\dfrac{1}{2(\lambda+1)}$ $\qquad \lambda\neq -1$

$\boxed{\cancel{\lambda=-1,\ (1)\ -1=0}}$

$\dfrac{2}{4(\lambda+1)^2}=2 \qquad (\lambda+1)^2=\dfrac{1}{4}$

$\lambda+1=\pm\frac{1}{2}$ $\quad \lambda=-1/2 \quad B(1,1)$

$\quad \lambda=-3/2 \quad C(-1,-1)$

$f(B)=1 < f(C)=5 \Rightarrow$ Weierstrass me dice en C máx global y B mín global EN LA FRONTERA.

$f(A)=1/2$

En S, C tiene que ser máx global. A mín global.

$$H_L\Big|_{\phi=0}=\begin{pmatrix} 2+2\lambda & 0 \\ 0 & 2+2\lambda \end{pmatrix} \qquad d^2L=2(\lambda+1)\left[dx^2+dy^2\right]=4(\lambda+1)dx^2$$

$\phi=0\Rightarrow d\phi=0 \qquad 2x\,dx+2y\,dy=0 \qquad x\,dx=-y\,dy \Rightarrow$ en C y B $\boxed{dx=-dy}$

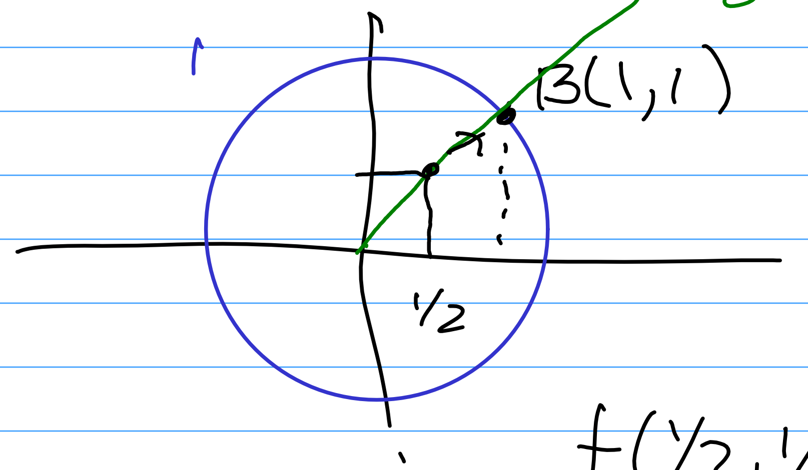
B) $\lambda = -1/2$ $\quad d^2L > 0$ mínimo local

c) $\lambda = -3/2$ $\quad d^2L < 0$ max local.

Por la frontera.

¿Qué es B?

$y = x \quad 0 < x < 1$

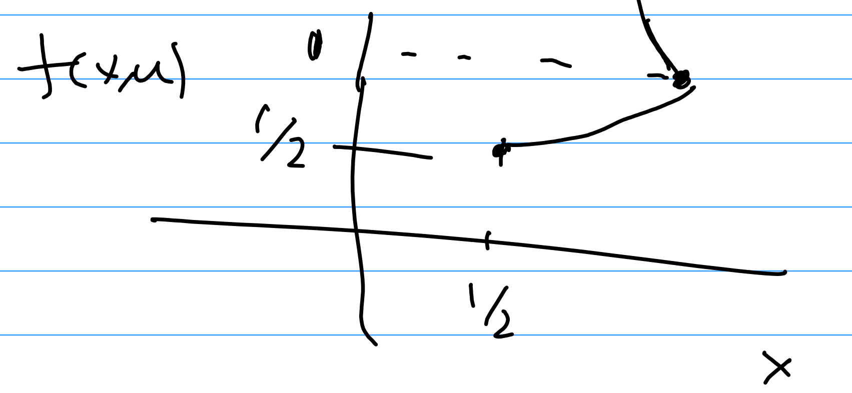


$f(B) = 1$

$f(x,x) = x^2 - x + x^2 - x + 1 = 2x^2 - 2x + 1$

$f' = 4x - 2 = 0 \qquad x = \frac{1}{2}$

$f'' = 4 \qquad x = 1/2$ mínimo.



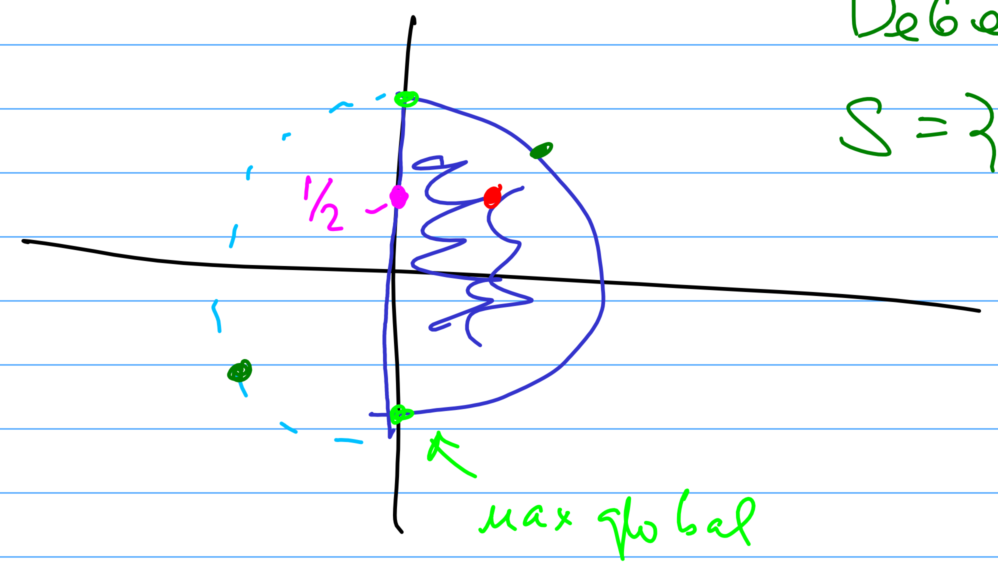
$f(1/2, 1/2) = 2 \cdot \frac{1}{4} - 1 + 1 = \frac{1}{2}$

$f(x,1) = x^2 - x + 1 = 1 + x(x-1) < 1 = f(1,1)$

$x < 1$

$f(x,y) = x^2 - x + y^2 - x + 1$ $S = \{ x^2+y^2 \leq 2,\ x \geq 0 \}$

Debería s $S = \{ x^2+y^2 \leq 2,\ x \leq 0 \}$



I) Extremos libres.

- $A(1/2, 1/2)$ mínimo local (global)

II) Extrs cond. $x^2+y^2=2$ $x \geq 0$

IIa) Extrs cond. $x^2+y^2=2$

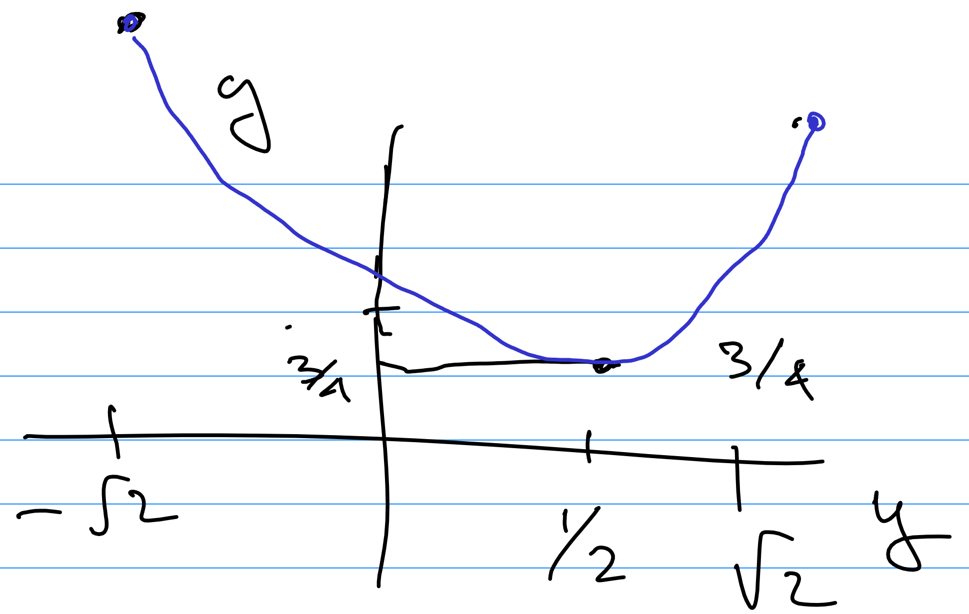
- $B(\cancel{1},1)$
- $C(\cancel{-1},-1)$

No es extremo en la región! No está en S

IIb) $x=0$, $\Rightarrow y^2 \leq 2$ $g = f(0,y)$ definida en $[-\sqrt{2}, \sqrt{2}]$

$g(y) = y^2 - y + 1$ $g'(y) = 0$ $2y - 1 = 0$ $y = 1/2$ $D(0, 1/2)$

$g'' = 2 > 0$ mínimo. $f(D) =$



$g(\sqrt{2}) = 2 - \sqrt{2} + 1$ max local !

$g(-\sqrt{2}) = 2 + \sqrt{2} + 1$ máx local.

Frontera
$x = 0$
$x^2 + y^2 \leq 2$

$$g(1/2) < g(\sqrt{2})$$
$$\|$$
$$f(1/2, 1/2) < f(0, 1/2) = 3/4$$
$$\equiv$$

$E(0, -\sqrt{2})$

$F(0, \sqrt{2})$

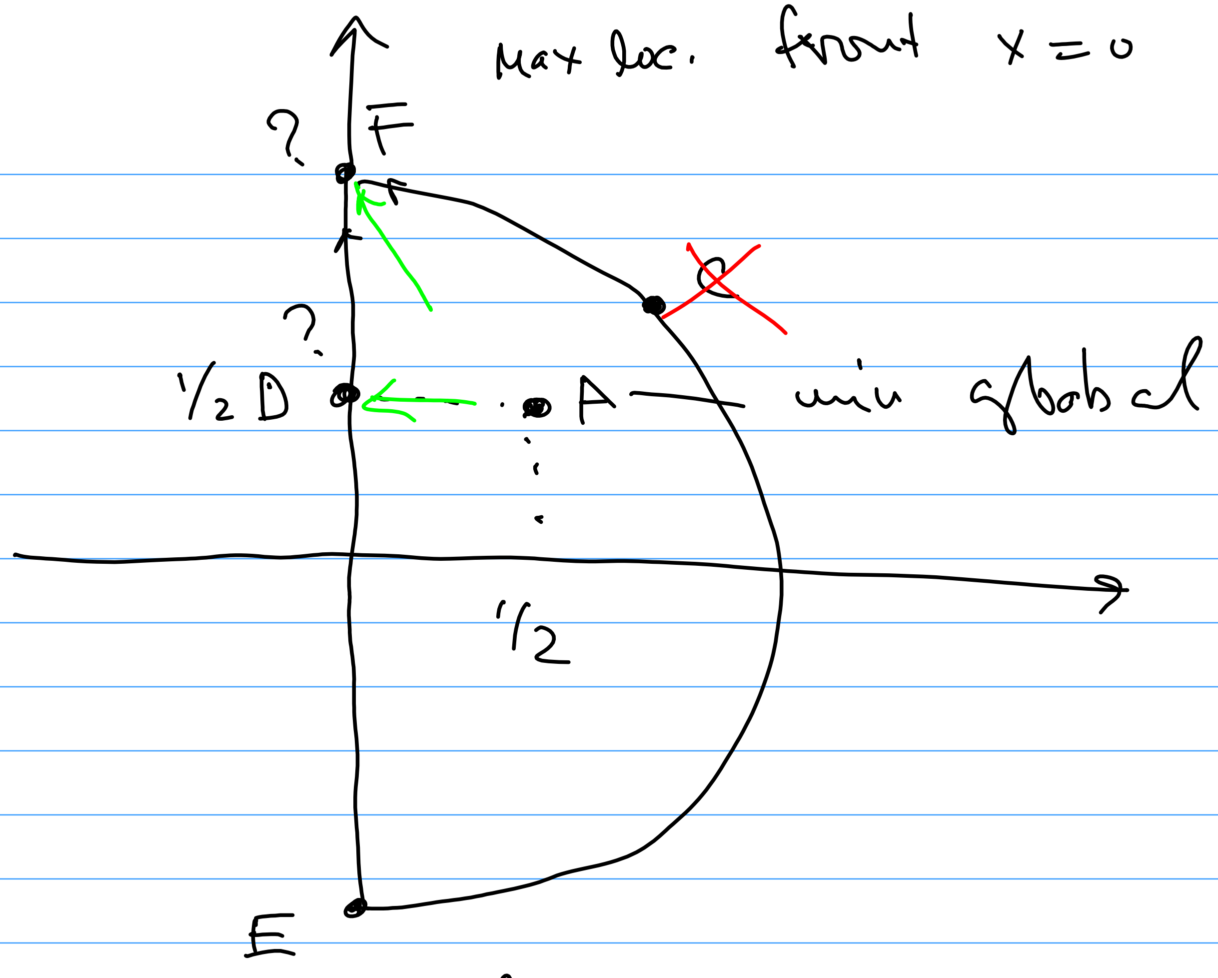
E es máx global de f en S.

$\Downarrow$ local en S

y A el min global de f en S

$\Downarrow$ local en S

max loc. front $x=0$ ¿en S?



max loc. $x=0$

max global en S

D) $f(x,1/2) = x^2 - x + 3/4 = \overset{<0}{x(x-1)} + 3/4 < 3/4$

$x>0$

D No es mín local en $\underline{\underline{S}}$

F) recta $y - \sqrt{2} = -\frac{\sqrt{2}}{a}x$

$f(x,y) = \dots$