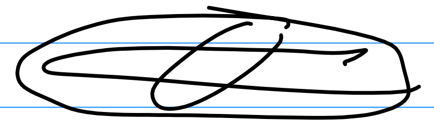
Sea $S \in \mathbb{R}^3$ *la superficie definida por* $x^2 + 4y^2 + 2z^2 = 9$.

Sea la función $f : S \mapsto \mathbb{R}$, $f(x, y, z) = x + \sqrt{3}y - z$ *definida sobre* S. *Encuentra todos los puntos críticos de* f *en* S *y decide si son extremos locales o puntos de silla.*



$$L = x + \sqrt{3}y - z + \lambda(x^2 + 4y^2 + 2z^2 - 9) \qquad x^2 + 4y^2 + 2z^2 = 9 \quad (4)$$

$$\frac{\partial L}{\partial x} = 1 + 2\lambda x = 0 \quad (1) \qquad \frac{\partial L}{\partial y} = \sqrt{3} + 8\lambda y = 0 \quad (2) \qquad \frac{\partial L}{\partial z} = -1 + 4\lambda z = 0 \quad (3)$$

~~Si $\lambda = 0$ $1 = 0$~~

$$x = -\frac{1}{2\lambda} \qquad y = \frac{-\sqrt{3}}{8\lambda} \qquad z = \frac{1}{4\lambda} \qquad \lambda \neq 0$$

Sust en (4)

$$\frac{1}{4\lambda^2} + \frac{\cancel{4} \cdot 3}{\underset{16}{\cancel{64}}\lambda^2} + \frac{2}{16\lambda^2} = 9 \Rightarrow \frac{4+3+2}{16\lambda^2} = 9 \qquad \lambda^2 = \frac{1}{16} \qquad \lambda = \pm 1/4$$

$$\lambda = \frac{1}{4} \quad A\left(-2, -\sqrt{3}/2, 1\right) \qquad \lambda = -1/4 \quad B\left(2, \sqrt{3}/2, -1\right)$$

2 ptos $\quad f(A) = -2-\frac{3}{2}-1 = -\frac{9}{2} \qquad f(B) = 2+\frac{3}{2}+1 = \frac{9}{2}$

$f(A) < f(B)$ T. Weierstrass nos dice que f alcanza máx y mín abs.

$\Rightarrow$ en A se alcanza el mín. abs. y en B el máx abs $\Rightarrow$

A – mín local y en B max abs.

$$H_L = \begin{pmatrix} 2\lambda & 0 & 0 \\ 0 & 8\lambda & 0 \\ 0 & 0 & 4\lambda \end{pmatrix} \qquad d^2L = 2\lambda\left(dx^2 + 4dy^2 + 2dz^2\right)$$

$2x\,dx + 8y\,dy + 4z\,dz = 0 \Rightarrow$ A) $-2\,dx - 2\sqrt{3}\,dy + 2dz = 0$ $\quad$ B) idem

$x\,dx + 4y\,dy + 2z\,dz \qquad dz = dx + \sqrt{3}\,dy$

$$d^2L = 2\lambda\left(\underbrace{dx^2 + 4dy^2 + 2(dx+\sqrt{3}dy)^2}_{>0}\right)$$

A $\lambda > 0 \Rightarrow d^2L > 0$ mínimo. $\qquad$ B $\lambda < 0 \Rightarrow d^2L < 0$ max

Sea $S \in \mathbb{R}^3$ la superficie definida por $x^2 + 4y^2 + 2z^2 = 9$.

dist $\begin{smallmatrix}\max\\ \min\end{smallmatrix} \Leftrightarrow \text{dist}^2$ $\begin{smallmatrix}\max\\ \min\end{smallmatrix}$

Encuentra, si existen, los puntos de la superficie S más próximos y más alejados del punto $(0,3,0)$.

$$f = \sqrt{x^2+(y-3)^2+z^2}$$

$$f(x,y,z) = x^2+(y-3)^2+z^2$$

$$L = x^2+(y-3)^2+z^2 + \lambda\left(x^2+4y^2+2z^2-9\right)$$

$$\frac{\partial L}{\partial x} = 2x+2x\lambda = 2x(\lambda+1) = 0 \quad (1)$$

$$\frac{\partial L}{\partial y} = 2(y-3)+8\lambda y = 2(4\lambda y+y-3) = 0 \quad (2)$$

$$\frac{\partial L}{\partial z} = 2z+4\lambda z = 2z(1+2\lambda) = 0 \quad (3)$$

$$x^2+4y^2+2z^2=9$$

(1) I $\lambda = -1$

II $x = 0$

I $x=-1$ (2) $-3y-3=0$ $y=-1$ (3) $z=0$

(4) $x^2+4=9$ $x^2=5$ $x=\pm\sqrt{5}$

$A(-\sqrt{5},-1,0)$ $B(\sqrt{5},-1,0)$

II $x=0$ (2) ? ~~IIa $\lambda=-1/2$~~ (2) $\rightarrow$ $y=-3$ (3) $36+\cancel{2z^2}=9$

IIb $z=0$ (3) $\rightarrow$ $4y^2=9$ $y=\pm 3/2$

$C(0,\frac{3}{2},0)$ $D(0,-3/2,0)$ $f=x^2+(y-3)^2+z^2$

$f(A)=5+16=21=f(B)$ $f(A)$ y $f(B)$ mayores valores

$\Downarrow$

$f(C)=\frac{9}{4}$ $f(D)=\frac{81}{4}=20+\frac{1}{4}$ Weierstrass que en A y B se alcanza los max. abs $\Rightarrow$ ptos de mayor distancias

$f(C)$ es el menor $\Rightarrow$ C es mín. abs

$\Downarrow$

pto de menor distancia.

? D ?

$$H_L = \begin{pmatrix} 2+2\lambda & 0 & 0 \\ 0 & 2+8\lambda & 0 \\ 0 & 0 & 2+4\lambda \end{pmatrix}$$

$\lambda = -\frac{3}{4}$, $(0, -3/2, 0)$

$$d L^2(D) = 2(\lambda+1)\,dx^2 + 2(1+\cancel{4\lambda})\,dy^2 + 2(1+2\lambda)\,dz^2$$

$$2x\,dx + 8y\,dy + 4z\,dz = 0 \Rightarrow dy = 0$$

$$2(1/4)\,dx^2 + 2(-1/2)\,dz^2 = \frac{1}{2}dx^2 - dz^2$$