$S = \{x^2+2y^2+4z^2=9\}$

1) $f: S \to \mathbb{R}$ $\quad f = x - y + \sqrt{3} z$ $\quad$ Pto críticos

$$L = x - y + \sqrt{3} z + \lambda (x^2+2y^2+4z^2-9)$$

$$\frac{\partial L}{\partial x} = 1 + 2\lambda x = 0 \quad (1) \qquad \frac{\partial L}{\partial y} = -1 + 4\lambda y = 0 \quad (2) \qquad \frac{\partial L}{\partial z} = \sqrt{3} + 8\lambda z = 0 \quad (3)$$

$$\frac{\partial L}{\partial \lambda} = x^2+2y^2+4z^2-9=0$$

(1) $x = -\frac{1}{2\lambda}$ $\quad$ (2) $y = \frac{1}{4\lambda}$

3) $z = \frac{-\sqrt{3}}{8\lambda}$ $\qquad \frac{1}{4\lambda^2} + \cancel{2}\frac{1}{\underset{8}{\cancel{16}}\lambda^2} + \frac{4\cdot 3}{64\cdot \lambda^2} = 9$

$$\left(\frac{1}{4} + \frac{1}{8} + \frac{3}{16}\right) = \lambda^2 \qquad \frac{1}{9}\,\frac{4+2+3}{16} = \frac{9}{9\cdot 16} = \frac{1}{16} \qquad \lambda = \pm\frac{1}{4}$$

$f = x - y + \sqrt{3} z$ $\quad$ cont en S. $\quad$ comp.

$\lambda = \frac{1}{4}$ $A\left(-2, 1, -\sqrt{3}/2\right)$ $\lambda = -\frac{1}{4}$ $B\left(2, -1, \sqrt{3}/2\right)$

$f(A) = -2-1-3/2 < f(B) = 2+1+3/2$

Por Weierstrass alcanza máx y mín. absolutos!

$\Downarrow$

en A mín abs. y B un máx absoluto

Deberes!! La segunda diferencial.

$$\frac{\partial L}{\partial x} = 1 + 2\lambda x = 0 \qquad \frac{\partial L}{\partial y} = -1 + 4\lambda y = 0 \qquad \frac{\partial L}{\partial z} = \sqrt{3} + 8\lambda z = 0$$

$$\lambda = \frac{1}{4} \quad A\left(-2, 1, -\sqrt{3}/2\right) \qquad \lambda = -\frac{1}{4} \quad B\left(2, -1, \sqrt{3}/2\right)$$

$$f(A) = -2 - 1 - 3/2 \quad < \quad f(B) = 2 + 1 + 3/2$$

$$H_L = \begin{pmatrix} 2\lambda & 0 & 0 \\ 0 & 4\lambda & 0 \\ 0 & 0 & 8\lambda \end{pmatrix} \qquad d^2L = 2\lambda\left(dx^2 + 2dy^2 + 4dz^2\right)$$

$$d\phi = 0 \qquad 2x\,dx + 4y\,dy + 8z\,dz = 0$$

$$A) \quad -2dx + 2dy - 2\sqrt{3}dz = 0 \qquad B) \quad 2dx - 2dy + 2\sqrt{3}dz = 0$$

$$dy = (dx + \sqrt{3}dz) \Rightarrow \text{sust. en } d^2L$$

$$d^2L = 2\lambda\left(dx^2 + 2(dx + \sqrt{3}dz)^2 + 4dz^2\right) \quad \geq 0 \qquad = 0 \iff dx = dz = 0$$

$d^2L > 0$ $\quad \lambda > 0 \quad$ pto A mínimo local

$d^2L < 0$ $\quad \lambda < 0 \quad$ pto B max local

2) Encuentra los ptos más lejanos y cercanos de S al $(0,0,3)$!

$f(x,y,z) = \cancel{\sqrt{x^2+y^2+z^2}}$ $\Rightarrow$ distancias $\Rightarrow$ distancias al cuadrado!

$f(x,y,z) = x^2+y^2+(z-3)^2$ $\qquad \phi = x^2+2y^2+4z^2-9 = 0 \quad (4)$

$L = x^2+y^2+(z-3)^2 + \lambda(x^2+2y^2+4z^2-9) = 0$

$\frac{\partial L}{\partial x} = 2x + 2x\lambda = 0 \quad (1) \quad 2x(\lambda+1) = 0$ $\quad \to \lambda = -1 \quad \text{I}$ $\quad \to x = 0 \quad \text{II}$

$\frac{\partial L}{\partial y} = 2y + 4\lambda y = 0 \quad (2) \quad 2y(2\lambda+1) = 0$

$\frac{\partial L}{\partial z} = 2(z-3) + 8\lambda z = 0 \quad (3) \quad z-3+4\lambda z = 0$

I) (2) $y = 0$ $\qquad$ (3) $z-3-4z = 0$ $\qquad z = -1$ $\qquad$ (4) $x^2+4-9 = 0$

$x = \pm\sqrt{5}$

$A(\sqrt{5}, 0, -1) \quad B(-\sqrt{5}, 0, -1) \qquad \lambda = -1$

II $\quad x=0 \qquad (2) \quad 2y(2\lambda+1)=0$

IIa $\quad \cancel{\lambda=-1/2}$

IIb $\quad y=0$

IIa) $\quad \lambda=\cancel{-}1/2 \qquad z-3-2z=0 \qquad -z-3=0 \qquad z=-3$

(4) $\quad 2y^2 \cancel{+36}=9$

IIb) $\quad x=0, \quad y=0 \qquad 4z^2=9 \qquad z=\pm 3/2$

$C(0,0,3/2) \qquad D(0,0,-3/2)$

$4\lambda-1=0 \qquad \lambda=1/4$

$z-3+4\lambda z=0 \qquad -\frac{3}{2}+4\cdot\frac{3}{2}\lambda=0$

$-\frac{3}{2}-\frac{6}{2}-4\frac{3}{2}\lambda=0 \qquad \frac{3}{2}+2\lambda=0 \qquad \lambda=-3/4$

$A(\sqrt{5},0,-1) \quad B(-\sqrt{5},0,-1) \qquad \lambda=-1$

$f(x,y,z)=x^2+y^2+(z-3)^2$

$f(A) = 5 + 16 = 21 = f(B)$

$f(C) = \frac{9}{4}$ $\quad f(D) = \frac{81}{4} < 21$ $\Rightarrow$

A, B don se alcanza el max. absoluto!!

C donde se alcanza el mí absoluto

¿ D ??

Hessiano!!

Debes H_L ...