$f(x,y) = x^4 + y^4 + 6x^2y^2 + 8x^3$

$\frac{\partial f}{\partial x} = 4x^3 + 12xy^2 + 24x^2 = 0 = 4x(x^2 + 3y^2 + 6x) = 0$ I

$\frac{\partial f}{\partial y} = 4y^3 + 12x^2y = 0 = 4y(y^2 + 3x^2) = 0$ II

1) $A(0,0)$ II si $x \neq 0 \Rightarrow y = 0.$ Si uso I $x^2 + 6x = 0$

$x(x+6) = 0 \Rightarrow x = -6$

2) $B(-6,0)$

$\frac{\partial^2 f}{\partial x^2} = 12x^2 + 12y^2 + 48x$ $\frac{\partial^2 f}{\partial y^2} = 12y^2 + 12x^2$ $\frac{\partial f}{\partial x \partial y} = 24xy$

$H_f(A) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ $H_f(B) = \begin{pmatrix} 12 \cdot 36 - 48 \cdot 6 & 0 \\ 0 & >0 \end{pmatrix} = \begin{pmatrix} >0 & 0 \\ 0 & >0 \end{pmatrix}$

definida positiva min.

$$x^3-y^3=(x-y)(x^2+xy+y^2)$$

$$f(x,y)=x^4+y^4+(x+y)^2+2$$

$$\frac{\partial f}{\partial x}=0=4x^3+2(x+y)=0 \qquad \mathrm{I}$$

$$\frac{\partial f}{\partial y}=0=4y^3+2(x+y)=0 \qquad \mathrm{II}$$

$\mathrm{I}-\mathrm{II}$ $\qquad 4x^3-4y^3=0$

$$x^3=y^3 \Leftrightarrow x=y$$

$$4x^3+4x=0 \qquad 4x(x^2+1)=0$$

$A(0,0)$, $B(\cancel{1},\cancel{1})$, $C(\cancel{-1},\cancel{-1})$

$$\frac{\partial^2 f}{\partial x^2}=12x^2-2 \qquad \frac{\partial^2 f}{\partial y^2}=12y^2-2 \qquad \frac{\partial^2 f}{\partial x \partial y}=-2$$

$$H_f(A)=\begin{pmatrix}-2 & -2\\ -2 & -2\end{pmatrix} \qquad H_f(B)=\begin{pmatrix}10 & -2\\ -2 & 10\end{pmatrix}=H_f(C)$$

definida positiva

$$\det H_f(B)>0$$

$f = x^4 + y^4 + z^4 - 2(x^2 + z^2) + y^2$

$\frac{\partial f}{\partial x} = 4x^3 - 4x = 0 \qquad 4x(x^2-1) = 0 \qquad x=0, \quad x=1, \quad x=-1$

$\frac{\partial f}{\partial y} = 4y^3 + 2y = 0 \qquad 2y(2y^2+1) = 0 \qquad \Rightarrow \quad y = 0$

$\frac{\partial f}{\partial z} = 4z^3 - 4z = 0 \qquad 4z(z^2-1) = 0 \qquad z=0 \qquad z=1 \qquad z=-1$

A $(0,0,0)$ $(0,0,1)$ $(0,0,-1)$

B $(1,0,0)$ $(1,0,1)$ $(1,0,-1)$

$(-1,0,0)$ $(-1,0,1)$ $(-1,0,-1)$

$\frac{\partial^2 f}{\partial x^2} = 12x^2 - 4 \qquad \frac{\partial^2 f}{\partial y^2} = 12y^2 + 2, \quad \frac{\partial^2 f}{\partial z^2} = 12z^2 - 4$

$\frac{\partial^2 f}{\partial x \partial y} = 0 \qquad \frac{\partial^2 f}{\partial x \partial z} = 0 \qquad \frac{\partial^2 f}{\partial y \partial z} = 0$

$$f(x,y) = \begin{cases} xy\log(x^2+y^2) & (x,y)\neq(0,0) \\ 0 & (x,y)=(0,0) \end{cases}$$

$$\lim_{(x,y)\to 0} f(x,y) = 0$$

$$\left.\begin{array}{l} x = r\cos\varphi \\ y = r\sin\varphi \end{array}\right\} \Rightarrow f(r,\varphi) = r^2\sin\varphi\cos\varphi \cdot \log r^2$$

$$\underbrace{(r^2\log r^2)}_{\substack{\downarrow \\ 0 \\ r\to 0}} \underbrace{\sin\varphi\cos\varphi}_{\substack{\text{acotado} \\ \forall\varphi}} \to 0$$

$$\frac{\partial f}{\partial x}(x,y) = y\log(y^2+x^2) + \frac{2x^2y}{y^2+x^2} = 0$$

$$\frac{\partial f}{\partial y} = x\log(x^2+y^2) + \frac{2y^2x}{x^2+y^2} = 0$$

$$\frac{\partial f}{\partial x}(0,0) = \frac{\partial f}{\partial y}(0,0) = 0$$

(debes $\frac{\partial f}{\partial x}$ $\frac{\partial f}{\partial y}$ son cont.)

$A(0,0)$?

$$(x,y)\neq(0,0) \quad x\left((x^2+y^2)\log(x^2+y^2) + 2y^2\right) = 0 \qquad (1)$$

$$y\left((x^2+y^2)\log(x^2+y^2) + 2x^2\right) = 0 \qquad (2)$$

(1) $x=0$ $(y\neq 0)$ $\Rightarrow$ (1) se cumple!

(2) $x=0$ $y\neq 0$ $y^2\log y^2=0$ $y=\pm 1$

$A_1(0,1)$ $A_2(0,-1)$ $B_1(1,0)$ $B_2(-1,0)$

(2) $=0$ $y=0$ $x^2\log x^2=0 \Rightarrow x=\pm 1$

(1) (2) $\Rightarrow$ restando lo que está entre ( ) $\Rightarrow$ $2x^2-2y^2=0 \Rightarrow x^2=y^2$

$x\neq 0$ e $y\neq 0$ $x=\pm y$

(2) $\Rightarrow$ $2x^2\log 2x^2+2x^2=0$

$2x^2(1+\log 2x^2)=0$ $\log 2x^2=-1$ $2x^2=e^{-1}$ $\Rightarrow x=\pm\frac{1}{\sqrt{2e}}$

$C_1=\left(\frac{1}{\sqrt{2e}},\frac{1}{\sqrt{2e}}\right)$ $C_2=\left(\frac{-1}{\sqrt{2e}},-\frac{1}{\sqrt{2e}}\right)$ $D_1=\left(\frac{1}{\sqrt{2e}},-\frac{1}{\sqrt{2e}}\right)$ $D_2=\left(-\frac{1}{\sqrt{2e}},\frac{1}{\sqrt{2e}}\right)$

Hs