$f(x,y)=x^2+y^2$ si $\Phi(x,y)=x^2+3xy+y^2-4=0.$

$$L = x^2+y^2+\lambda(x^2+y^2+3xy-4)$$

$$(1)\quad \frac{\partial L}{\partial x} = 2x+\lambda(2x+3y)=0$$

$$(2)\quad \frac{\partial L}{\partial y} = 2y+\lambda(2y+3x)=0$$

$$(3)\quad \frac{\partial L}{\partial \lambda} = x^2+3xy+y^2-4=0$$

$(1)\,x-(2)\,y \qquad 2x^2+2\lambda x^2+3\cancel{\lambda}xy-(2y^2+2\lambda y^2+3\cancel{\lambda}xy)=0$

$$2(x^2-y^2)(\lambda+1)=0 \quad \left\{\begin{array}{ll} \lambda=\underline{\underline{-1}} & \mathrm{I} \\ x=y & \mathrm{II} \\ x=-y & \mathrm{III} \end{array}\right.$$

$\cancel{\lambda=-1,\ (1)\ 3y=0\quad y=0}$

$\cancel{(2)\ 3x=0\quad x=0\quad (3)\ -4=0}$

$\mathrm{II})\ x=y \qquad x^2+3x^2+x^2-4=0 \qquad 5x^2=4 \qquad x^2=4/5 \qquad x=\pm 2/\sqrt{5}$

$\mathrm{III}\ \ \cancel{x=-y} \qquad \cancel{x^2-3x^2+x^2-4=0} \Rightarrow -x^2-\cancel{4}=0$

$A(2/\sqrt{5},\,2/\sqrt{5}) \qquad B(-2/\sqrt{5},\,-2/\sqrt{5}) \qquad f(A)=f(B)=\frac{8}{5}$

$2+\lambda 2+3\lambda=0 \qquad 5\lambda+2=0 \qquad \lambda=-2/5$

Matriz

$$d^2L(x_0,y_0) = H_L = \begin{pmatrix} \frac{\partial^2 L}{\partial x^2} & \frac{\partial^2 L}{\partial x \partial y} \\ \frac{\partial^2 L}{\partial y \partial x} & \frac{\partial^2 L}{\partial y^2} \end{pmatrix} = \begin{pmatrix} 2+2\lambda & 3\lambda \\ 3\lambda & 2+2\lambda \end{pmatrix}$$

x, y no son indep.

$$d^2L = 2(\lambda+1)\,dx^2 + 2(\lambda+1)\,dy^2 + 6\lambda\,dx\,dy$$

No vale hacer:

$$\cancel{d^2L(A) = 2(1-2/5)\,dx^2 + 2(1-2/5)\,dy^2 - 6\cdot\frac{2}{5}\,dx\,dy}$$

$$\cancel{\begin{pmatrix} 6/5 & -6/5 \\ -6/5 & 6/5 \end{pmatrix}} \qquad \cancel{\frac{6}{5}dx^2 + \frac{6}{5}dy^2 - \frac{12}{5}dx\,dy}$$

$$\cancel{\frac{6}{5}(dx - dy)^2}$$

$$\Phi = x^2+y^2+3xy-4=0 \qquad d\Phi = (2x+3y)\,dx + (2y+3x)\,dy = 0$$

$$d\Phi(A) = d\Phi(B) = 0 \Rightarrow 5\,dx + 5\,dy = 0 \Rightarrow \boxed{dx = -dy}$$

$$d^2L = 4(\lambda+1)\,dx^2 - 6\lambda\,dx^2 \Big|_{\lambda=-2/5} = \frac{12}{5}dx^2 + \frac{12}{5}dx^2 = \frac{24}{5}dx^2 > 0$$

en A y B

Encontrar los extremos de la función $f(x,y) = x^2 + y^2$ con la condición de ligadura $(x-3)^2 + (y-4)^2 = a^2$, con $a = 10$ y $a = 2$.

$$L = x^2+y^2 + \lambda\left((x-3)^2+(y-4)^2-10^2\right)$$

(1) $\frac{\partial L}{\partial x} = 2x + 2\lambda(x-3) = 0$

(2) $\frac{\partial L}{\partial y} = 2y + 2\lambda(y-4) = 0$

(3) $\frac{\partial L}{\partial \lambda} = (x-3)^2 + (y-4)^2 - 10^2 = 0$

(1) $\to$ $x(\lambda+1) = 3\lambda \Rightarrow x = \frac{3\lambda}{\lambda+1}$

(2) $\to$ $y = \frac{4\lambda}{\lambda+1}$ $\quad \lambda \neq -1$

~~Si $\lambda = -1$ (1) $6=0$~~

Sus. (3) $\left(\frac{3\lambda}{\lambda+1} - 3\right)^2 + \left(\frac{4\lambda}{\lambda+1} - 4\right)^2 = 100$

$$\frac{\lambda}{\lambda+1} - 1 = \frac{\lambda - \lambda - 1}{\lambda+1} = \frac{-1}{\lambda+1}$$

$$\frac{9}{(\lambda+1)^2} + \frac{16}{(\lambda+1)^2} = 100 = \frac{25}{(\lambda+1)^2}$$

$$(\lambda+1)^2 = \frac{1}{4} \Rightarrow \lambda+1 = \pm 1/2$$

$\lambda = -3/2 \quad (9, 12)$ A

$\lambda = -1/2 \quad (-3, -4)$ B

$f(A) = 81 + 144 = 225$ $\qquad$ $f(B) = 9 + 16 = 25$

f es cont en la circunferencia que es un compacto $\Rightarrow$ f alcanza máx y mín absolutos. f es difble y tiene 2 pts críticos A y B $\Rightarrow$ $f(A) > f(B)$ $\Rightarrow$ A es max global y B es mín global $\Rightarrow$ A es max local y B mín. local!

$$* \quad H_L = \begin{pmatrix} 2+2\lambda & 0 \\ 0 & 2+2\lambda \end{pmatrix} \qquad d^2L = (2+2\lambda)dx^2 + (2+2\lambda)dy^2$$

Vamos a hacerlo MHL.

$d^2L(A) = 2(-1/2)\left(dx^2 - dy^2\right) < 0$ máx, $\quad d^2L(B) = dx^2 + dy^2 > 0$ mín

$d\phi = 0 \qquad 2(x-3)dx + 2(y-4)dy = 0 \qquad (x-3)dx + (y-4)dy = 0$

A) $(9,12)$ $\quad 6dx + 8dy = 0 \quad dy = -\frac{3}{4}dx$ $\qquad$ B $(-3,-4)$ $\quad -6dx - 8dy = 0 \quad dy = -\frac{3}{4}dy$

$$d^2L = 2(\lambda+1)dx^2\left[1+\tfrac{9}{16}\right]$$

$\vee$
0

A $\lambda = -3/2$ $d^2L < 0$ max

B $\lambda = -1/2$ $d^2L > 0$ mín

$f(x,y) = xy$ si $x^2+y^2=1$

$L = xy + \lambda(x^2+y^2-1) = 0$

(1) $\dfrac{\partial L}{\partial x} = y + 2x\lambda = 0$

(2) $\dfrac{\partial L}{\partial y} = x + 2y\lambda = 0$

(3) $\dfrac{\partial L}{\partial \lambda} = x^2+y^2-1 = 0$

$(1)\cdot y - (2)\cdot x = y^2 + \cancel{2xy\lambda} - x^2 - \cancel{2xy\lambda} = 0$

$x^2 = y^2$ $\begin{cases} x = y & \mathrm{I} \\ x = -y & \mathrm{II} \end{cases}$

I usar (3) $\Rightarrow$ $2x^2 = 1$ $x = \pm \frac{1}{\sqrt{2}}$

$A(1/\sqrt{2}, 1/\sqrt{2})$ $B(-1/\sqrt{2}, -1/\sqrt{2})$

II $\Rightarrow x = \pm 1/\sqrt{2}$ $C(1/\sqrt{2}, -1/\sqrt{2})$ $D(-1/\sqrt{2}, 1/\sqrt{2})$

$f(A) = \frac{1}{2} = f(B)$ $f(C) = -\frac{1}{2} = f(D)$

A y B máx globales $\Rightarrow$ locales y C y D mínimos globales $\Rightarrow$ locales

(1) A y B $1+2\lambda=0 \Rightarrow \lambda = -1/2$, C y D $1-2\lambda=0$ $\lambda = 1/2$

$$H_L = \begin{pmatrix} 2\lambda & 1 \\ 1 & 2\lambda \end{pmatrix}$$

$d^2L = 2\lambda dx^2 + 2\lambda dy^2 + 2dxdy$

$d\phi = 0 \quad 2xdx + 2ydy = 0 \Rightarrow xdx + ydy = 0$

A y B $\quad dx = -dy \Rightarrow d^2L = -dx^2 - dx^2 - 2dx^2 = -4(dx)^2 < 0$ max.

C y D $\quad dx = dy \Rightarrow d^2L = dx^2 + dx^2 + 2dx^2 = 4(dx)^2 > 0$ min.