$F(x,y)=0 = f(x)-y = 0 \qquad F: \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n \qquad f: \mathbb{R}^n \to \mathbb{R}^n \qquad y \in \mathbb{R}^n$

$f \in C^p(A) \qquad x_0 \in A.$

$A \supset U(x_0) \qquad x \to f(x) \qquad f(x) \subset V(f(x_0))$

$(x_0, y_0) \in \mathbb{R}^n \times \mathbb{R}^n$

? $x = g(y)$

1º $F(x_0,y_0)=0 \Rightarrow f(x_0)-y_0 = 0 \qquad \forall x_0 \in A$ (en particular en $U(x_0)$)

2º $F \in C^p(A) \Rightarrow f \in C^p(A)$

3º $$F'_x(x_0,y_0) = \begin{pmatrix} \frac{\partial F_1}{\partial x_1} & \cdots & \frac{\partial F_1}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial F_n}{\partial x_1} & & \frac{\partial F_n}{\partial x_n} \end{pmatrix} \qquad \det F'_x(x_0,y_0) \neq 0$$

$\Rightarrow$ T.F.I. $\exists\ x = g(y)$ t.q. $F(g(y),y)=0 \quad \forall y \in I_y = V(y_0) \qquad g \in C^p(V(y_0))$

$f(g(y)) = y \quad \forall y \in V(y_0) \Rightarrow g = f^{-1} \qquad f: U(x_0) \to V(x_0) \Rightarrow g: V(x_0) \to U(x_0)$

$$Dg = Df^{-1} = -(F_x')^{-1}(F_y') = -\underline{(f'(x))^{-1}}(-I)$$

$$= (f'(x))^{-1}$$

$$F(x,y) = f(x) - y = \begin{pmatrix} f_1 - y_1 \\ f_2 - y_2 \\ \vdots \\ f_n - y_n \end{pmatrix}$$

$$F_y' = \begin{pmatrix} \frac{\partial F_1}{\partial y_1} & \cdots & \frac{\partial F_1}{\partial y_n} \\ \vdots & & \vdots \\ \frac{\partial F_n}{\partial y_1} & & \frac{\partial F_n}{\partial y_n} \end{pmatrix} = \begin{pmatrix} -1 & 0 & \cdots & 0 \\ 0 & -1 & \ddots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -1 \end{pmatrix}$$

$$= -I_n$$

$$F_x' = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix} = f'(x)$$

$$\Rightarrow \quad Df^{-1}(x) = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & \\ \frac{\partial f_n}{\partial x} & & \frac{\partial f_n}{\partial x_n} \end{pmatrix}^{-1}$$

Sea $f : \mathbb{R}^2 \longmapsto \mathbb{R}^2$, definida por $f(u, v) = (e^u + e^v, e^u - e^v)$.

$\exists ?$ $f^{-1}(x,y)$

$u = u(x,y)$

$v = v(x,y)$

$$f(u,v) = \begin{pmatrix} x(u,v) \\ y(u,v) \end{pmatrix} = \begin{pmatrix} e^u + e^v \\ e^u - e^v \end{pmatrix} = \begin{pmatrix} f_1(u,v) \\ f_2(u,v) \end{pmatrix}$$

$f(u,v)$ tiene inversa

$f : \mathbb{R}^2 \to \mathbb{R}^2$ $\quad x(u,v) > 0 \quad y(u,v) \in \mathbb{R}$

$$f'(u,v) = Df(u,v) = \begin{pmatrix} \frac{\partial f_1}{\partial u} & \frac{\partial f_1}{\partial v} \\ \frac{\partial f_2}{\partial u} & \frac{\partial f_2}{\partial v} \end{pmatrix} = \begin{pmatrix} e^u & e^v \\ e^u & -e^v \end{pmatrix}$$

$\det Df(u,v) = -2e^{u+v} \neq 0 \quad \forall u,v \in \mathbb{R} \Rightarrow$ T. F. Inversa.

$\exists$ u,v $\quad h(x,y)$ es la inversa de $f(u,v)$ $\quad h(x,y) = \begin{pmatrix} u(x,y) \\ v(x,y) \end{pmatrix}$

- f es invertible localmente $\forall (u,v) \in \mathbb{R}^2$

$$Dh(x,y) = (Df)^{-1} = \frac{1}{-2e^{u+v}} \begin{pmatrix} -e^{v} & -e^{v} \\ -e^{u} & e^{u} \end{pmatrix} = \begin{pmatrix} e^{-u}/2 & e^{-u}/2 \\ e^{-v}/2 & -e^{-v}/2 \end{pmatrix}$$

$$\begin{pmatrix} e^{u} & e^{u} \\ e^{v} & -e^{v} \end{pmatrix}$$

- $\exists$? inversa global.

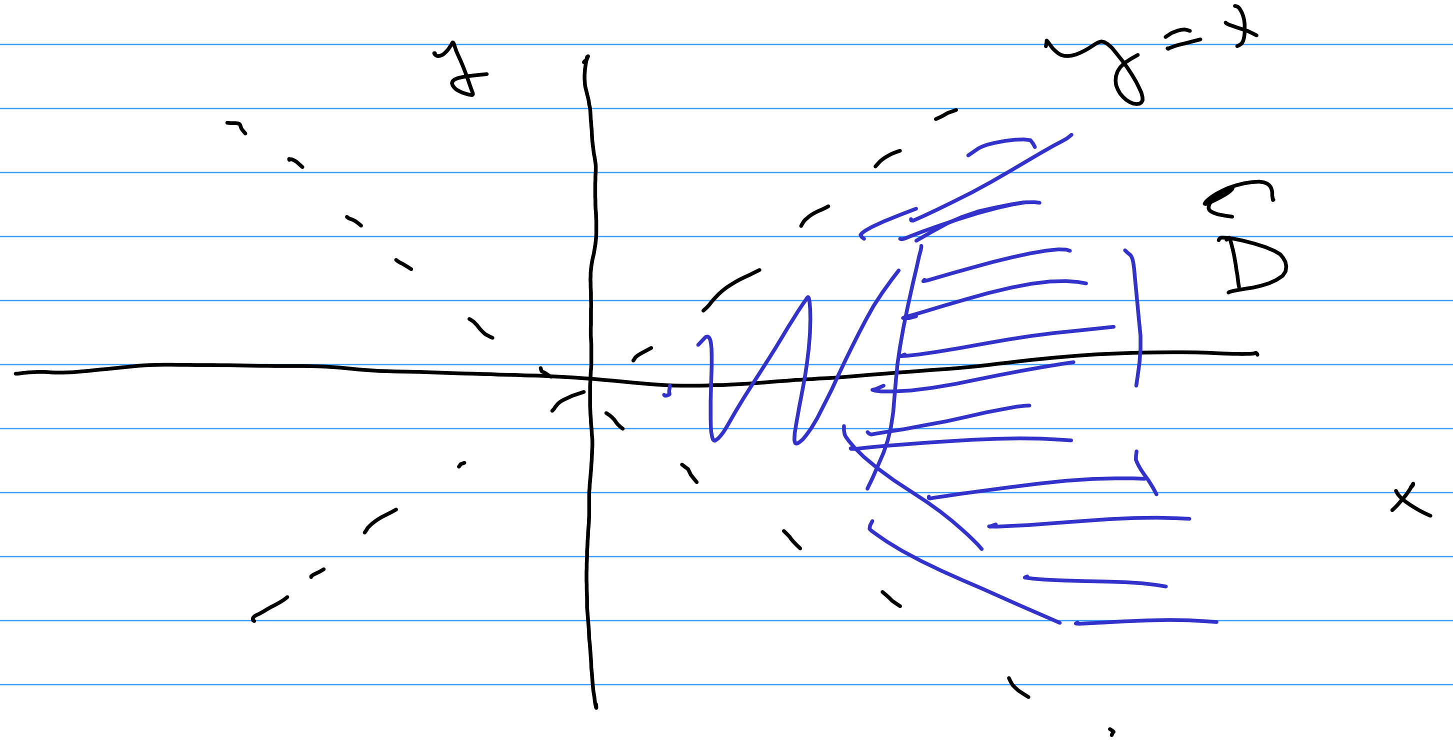
$$\begin{pmatrix} e^{u}+e^{v} \\ e^{u}-e^{v} \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$x = e^{u} + e^{v}$

$y = e^{u} - e^{v}$

$x+y = 2e^{u} > 0 \qquad e^{u} = \frac{x+y}{2} \Rightarrow u = \log \frac{x+y}{2}$ (1)

$x-y = 2e^{v} > 0 \qquad v = \log \frac{x-y}{2}$ (2)



$$\left.\begin{array}{ll} (2) \to & y < x \\ (1) & y > -x \end{array}\right\} D.$$

$h: D \Rightarrow \mathbb{R}^2$

$$h(x,y) = \begin{pmatrix} \log\left(\frac{x+y}{2}\right) \\ \log\left(\frac{x-y}{2}\right) \end{pmatrix} \qquad Dh(x,y) = \begin{pmatrix} \frac{\partial h_1}{\partial x} & \frac{\partial h_1}{\partial y} \\ \frac{\partial h_2}{\partial x} & \frac{\partial h_2}{\partial y} \end{pmatrix} = \begin{pmatrix} \frac{1}{x+y} & \frac{1}{x+y} \\ \frac{1}{x-y} & -\frac{1}{x-y} \end{pmatrix}$$

$$\frac{1}{\frac{x+y}{\cancel{2}}} \; \cancel{1/2} \qquad = \begin{pmatrix} \frac{e^{-u}}{2} & \frac{e^{-u}}{2} \\ e^{-v}/2 & -e^{-v}/2 \end{pmatrix}$$

Sea $f(x,y) = (e^x \cos y, e^x \operatorname{sen} y)$.

a) Probar que f es localmente invertible, pero no globalmente, aunque su jacobiano es distinto de 0 siempre.

b) Comprobar que si $U = \{(x,y) \in \mathbb{R}^2 : -\pi \leq y < \pi\}$ entonces $f_{|U}$ es invertible, siendo su inversa continua.

$$f(x,y) = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} e^x \cos y \\ e^x \operatorname{sen} y \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix}$$

Debemos: Calcular $(Df)^{-1} \Rightarrow$ Calcular Df^{-1}

$$Df = f' = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{pmatrix} = \begin{pmatrix} e^x \cos y & -e^x \operatorname{sen} y \\ e^x \operatorname{sen} y & e^x \cos y \end{pmatrix}$$

$$\det f' = e^{2x} \cos^2 y + e^{2x} \operatorname{sen}^2 y = e^{2x} \neq 0 \quad \forall x \in \mathbb{R}$$

$$f = \begin{pmatrix} e^x \cos y \\ e^x \operatorname{sen} y \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix} \qquad \begin{matrix} e^x \cos(y + 2\pi k) = e^x \cos y \\ e^x \operatorname{sen}(y + 2k\pi) = e^x \operatorname{sen} y \end{matrix} \qquad \begin{pmatrix} u_1 \\ v_1 \end{pmatrix} = \begin{pmatrix} u_2 \\ v_2 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} t \\ \xi \end{pmatrix}$$

$$e^x \cos y = e^t \cos \xi \qquad e^{2x} \cos^2 y = e^{2t} \cos^2 \xi$$

$$e^x \operatorname{sen} y = e^t \operatorname{sen} \xi \qquad e^{2x} \operatorname{sen}^2 y = e^{2t} \operatorname{sen}^2 \xi$$

$$\Downarrow \qquad e^{2x} = e^{2t} \Rightarrow x = t$$

$$\begin{cases} \cos y = \cos \xi \\ \operatorname{sen} y = \operatorname{sen} \xi \end{cases} \Rightarrow \cos y = \cos(\overset{\xi}{y + 2\pi}) \iff y = \xi$$

necesito restringirme a la banda $y \in (-\pi, \pi]$

$$f : \mathbb{R}^2 \to \mathbb{R}^2 \qquad f|_D : \mathbb{R} \times \{(-\pi, \pi]\}.$$

$$f = \begin{pmatrix} e^x \cos y \\ e^x \operatorname{sen} y \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix}$$

$$\begin{aligned} e^x \cos y &= u \qquad (1)\ \ e^{2x} = u^2+v^2 \\ e^x \operatorname{sen} y &= v \qquad (2)\ \ \tan y = \frac{v}{u} \end{aligned}$$

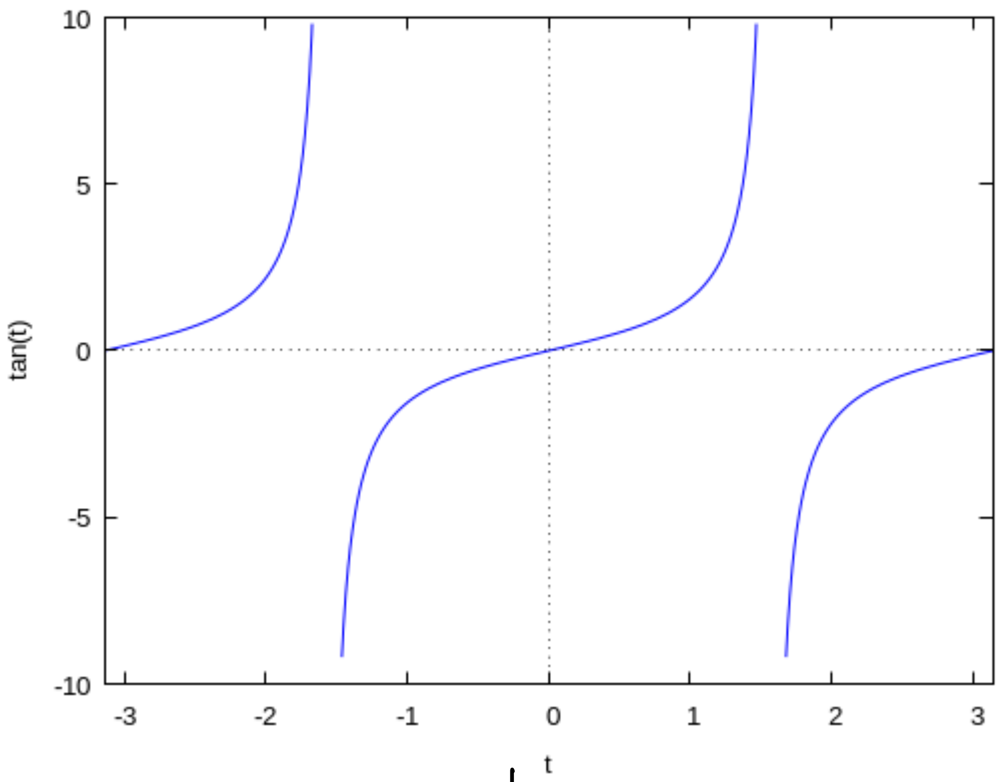
$$y \in (-\pi, \pi]$$

$$(1) \to x = \frac{\log(u^2+v^2)}{2} \qquad uv \neq 0$$

$$(2) \to u \neq 0$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = h(u,v) : \mathbb{R}^2 \to \mathbb{R} \times (-\pi,\pi]$$

$$\begin{cases} u>0 \quad v \geq 0 & y = \operatorname{artg}\left|\frac{v}{u}\right| \\ u<0 \quad v<0 & y = -\pi + \operatorname{artg}\left|\frac{v}{u}\right| \\ u<0 \quad v \geq 0 & y = \pi - \operatorname{artg}\left|\frac{u}{v}\right| \\ u>0 \quad v \leq 0 & y = -\operatorname{artg}\left|\frac{u}{v}\right| \end{cases}$$



$$u=0 \Rightarrow y = \pm\pi/2 \left\langle \begin{array}{lll} y=\pi/2 & e^x = v > 0 & x = \log v \\ y = -\pi/2 & e^x = -v > 0 & x = \log(-v) \end{array} \right\} \log|v|$$