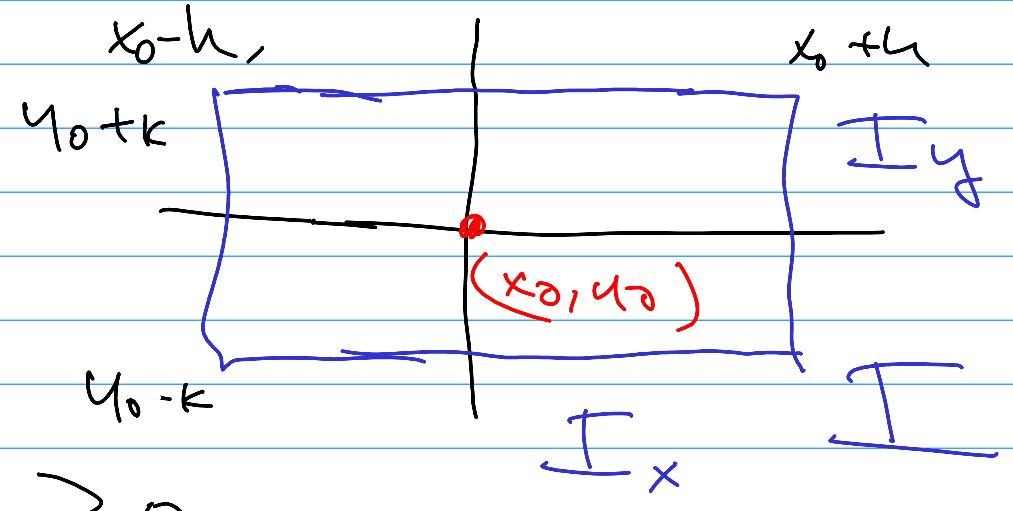
$F(x,y)=0 \qquad x\in\mathbb{R}^n \qquad y\in\mathbb{R} \qquad y=f(x)$

I) $F'_y(x_0,y_0)>0 \qquad F'_y$ en cont $\qquad F\in C^{(p)}(A) \quad p\geq 1$

$\exists I$ donde $F'_y(x,y)$ es >0



$h(y)=F(x,y) \qquad h'(y)>0$ en I_y

$$0>F(x_0,y_0-k)<F(x_0,y_0)<F(x_0,y_0+k)>0$$
$$F(x_0,y_0) = 0$$

F es cont en A, F es cont en I.

$\exists I_x \quad 0>F(x,y_0-k) < F(x_0,y_0) < F(x,y_0+k)>0$
$$F(x_0,y_0) = 0$$

Sea $\forall x\in I_x \quad h(y)$ cambia de signo en $[y_0-k,y_0+k] \qquad h$ es cont.

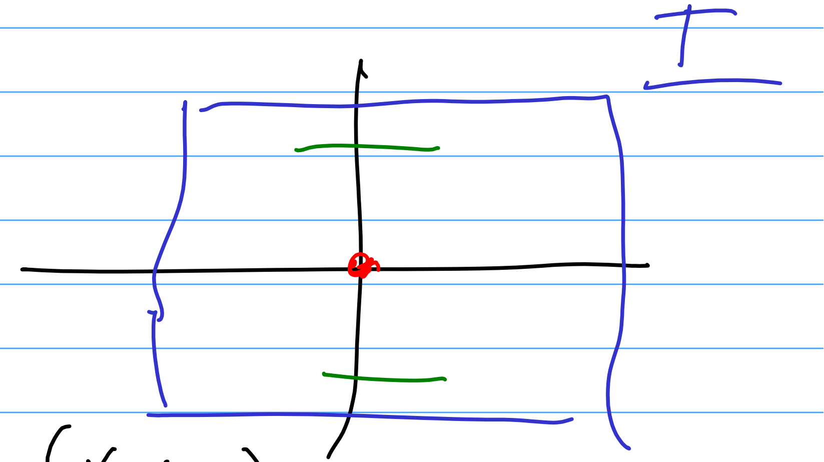
T. Bolzano $\qquad \exists! y\in(y_0-k,y_0+k)=I_y \qquad h(y)=0 \qquad h'(y)>0$

$\forall x \in I_y$ existe un único $y = f(x)$ $\qquad F(x,y) = 0$

$\exists f: I_x \to I_y$, $F(x, f(x)) = 0$ $\quad \forall x \in I_x$.

II) $f \in C^0(I_x)$ $\quad \forall \varepsilon > 0$ $\exists \delta > 0$ tq. $\|x - x_0\| < \delta \Rightarrow |f(x) - f(x_0)| < \varepsilon$

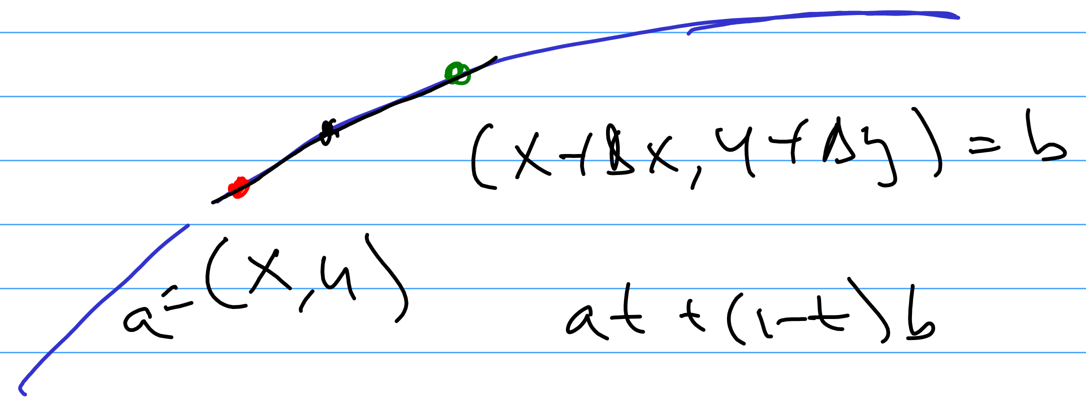
$\forall \varepsilon > 0$ $\quad B_\varepsilon(y_0) \subset I_y$ $\quad$ Supongo $\exists \delta > 0$ tq. $(\delta < k)$



En los extremos de $[y_0 - \delta, y_0 + \delta] \subset I_y$ $\quad h(y)$ cambia de signo. $\quad f(x) \subset B_{y_0}(\varepsilon)$
$x \in B_{x_0}(\delta)$

$\Rightarrow f$ es cont en un entorno de (x_0, y_0)

III $\quad f \in C^1(I_x)$ $\quad \frac{\partial f}{\partial x_i} = \ldots$ $\quad (x, y)$ $\quad (x + \Delta x, y + \Delta y)$



$y = f(x)$, $\quad f(x + \Delta x) = y + \Delta y$

$\Delta y = f(x + \Delta x) - f(x) \xrightarrow{\Delta x \to 0} 0$

$\to$ convexo.

F definida en $I \subset A$ $\qquad F(x,y) = 0$

$$0 = F(x+\Delta x, y+\Delta y) - F(x,y) \overset{\text{T.V.M. f v.v.}}{=} DF(\underbrace{x+\xi\Delta x, y+\xi\Delta y}_{\eta})\begin{pmatrix}\Delta x\\ \Delta y\end{pmatrix} \quad \xi\in(0,1)$$

$$\begin{pmatrix}\frac{\partial F}{\partial x_1}(\eta) & \frac{\partial F}{\partial x_2}(\eta) & \cdots & \frac{\partial F}{\partial x_n}(\eta) & \frac{\partial F}{\partial y}(\eta)\end{pmatrix}\begin{pmatrix}\Delta x_1\\ \Delta x_2\\ \vdots\\ \Delta x_n\\ \Delta y\end{pmatrix} = 0$$

$\Delta x = h e_i \qquad i = 1 \ldots n \qquad$ base canónica $\mathbb{R}^n$

$$e_i = \begin{pmatrix}0\\0\\ \vdots\\ 1\\ 0\\ \vdots\\ 0\end{pmatrix} \longrightarrow i \text{ posic.}$$

$$\frac{\partial F}{\partial x_i}(x+\xi h e_i, y+\xi\Delta y)\,h + \frac{\partial F}{\partial y}(x+\xi h e_i, y+\xi\Delta y)\,\Delta y = 0$$

$$\frac{\partial F}{\partial y}(x+\xi h e_i, y+\xi \Delta y)\frac{\Delta y}{h} = -\frac{\partial F}{\partial x_i}(x+\xi h e_i, y+\xi \Delta y)$$

$\Delta x = h e_i$
$\Delta y \to 0$
si $\Delta x \to 0$

$h \to 0$

$$\underbrace{\frac{\partial F}{\partial y}(x,y)}_{\neq 0} \lim_{h\to 0} \frac{\Delta y}{h} = -\frac{\partial F}{\partial x_i}(x,y)$$

$h \to 0$

$$\lim_{h\to 0} \frac{\Delta y}{h} = -\frac{\frac{\partial F}{\partial x_i}(x,y)}{\frac{\partial F}{\partial y}(x,y)}$$

$$\frac{\Delta y}{h} = \frac{f(x+he_i) - f(x)}{h} \to \frac{\partial f}{\partial x_i}$$

$$\frac{\partial f}{\partial x_i}(x) = -\left(F'_y(x,y)\right)^{-1} F'_{x_i}(x,y) = -\left(F'_y(x,f(x))\right)^{-1} F'_{x_i}(x,f(x))$$

$\forall i = 1,\dots,n$ $\quad \frac{\partial f}{\partial x_i}$ es cont en I_x $\Rightarrow f \in C^1$