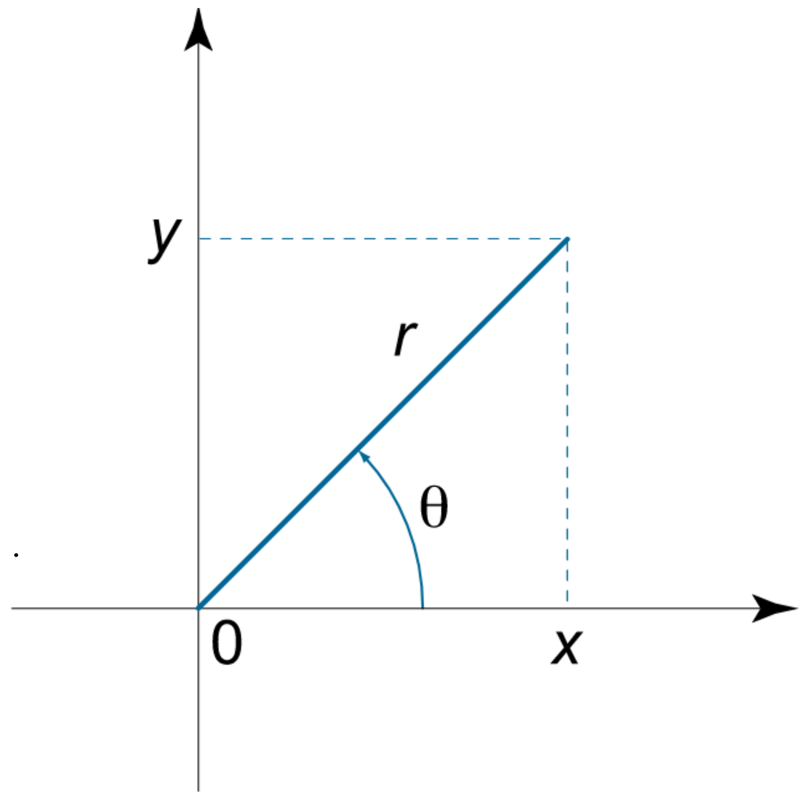
$$x = r\cos\theta, \qquad y = r\sin\theta$$

$$f(r,\theta) = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} f_1(r,\theta) \\ f_2(r,\theta) \end{pmatrix} \qquad r \geq 0 \quad \theta \in \mathbb{R}.$$



$$f : \underbrace{[0,\infty) \times \mathbb{R}}_{D} \to \mathbb{R}^2 \qquad f \in C^\infty(D)$$

$$Df(r,\theta) = \begin{pmatrix} \frac{\partial f_1}{\partial r} & \frac{\partial f_1}{\partial \theta} \\ \frac{\partial f_2}{\partial r} & \frac{\partial f_2}{\partial \theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & -r\,\mathrm{sen}\,\theta \\ \mathrm{sen}\,\theta & r\cos\theta \end{pmatrix}$$

$$\det Df = r\cos^2\theta + r\,\mathrm{sen}^2\theta = r \neq 0$$

localmente invertible. $(r_0, \varphi_0, x_0, y_0)$

$$r^2 = x^2 + y^2 \Rightarrow r = \sqrt{x^2+y^2}$$

$x \neq 0$ $\quad \tan\theta = y/x \quad \theta \in (-\pi, \pi]$

$\theta \to \theta + 2\pi \quad x \to x \quad y \to y$

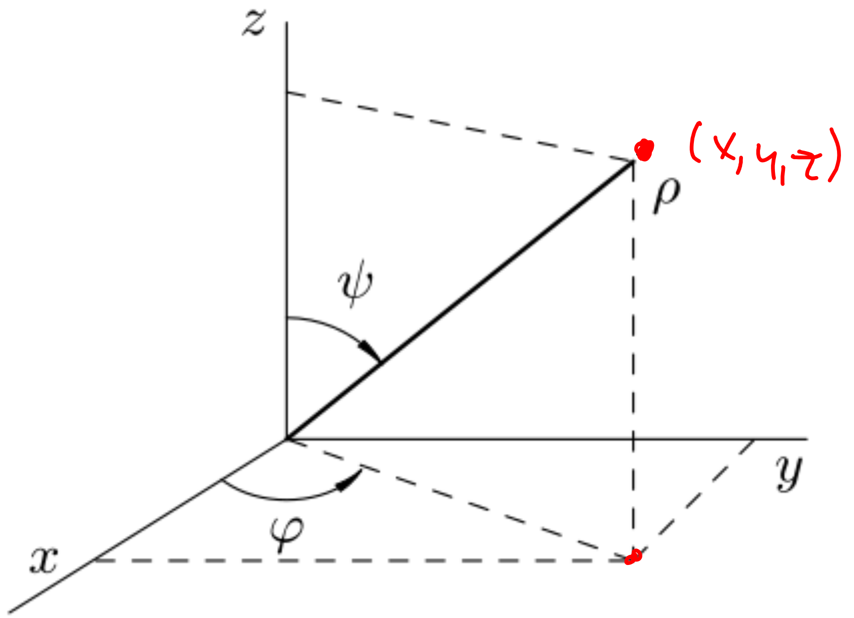
$x \neq 0$
$$\begin{cases} \theta = \operatorname{arctg} y/x, & x>0 \;\; y \geq 0 \qquad x<0 \;\; y \geq 0 \\ \theta = -\pi + \operatorname{arctg} y/x, & x<0 \;\; y<0 \\ \theta = \pi + \operatorname{arctg} y/x & x>0, \;\; y \leq 0 \end{cases}$$

$$x > 0 \Rightarrow \quad \theta = \begin{cases} \pi/2 & y = r \\ -\pi/2 & y = -r \end{cases} \quad \to \quad r = \sqrt{x^2+y^2}$$

$$z = \rho \cos \psi,$$
$$y = \rho \sin \psi \sin \varphi,$$
$$x = \rho \sin \psi \cos \varphi.$$

$$f(\rho, \psi, \varphi) = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$\rho \geq 0 \qquad \psi, \varphi \in \mathbb{R}$



$f : \underbrace{[0,\infty) \times \mathbb{R} \times \mathbb{R}}_{D} \to \mathbb{R}^3 \qquad f \in C^\infty(D)$

$(\rho, \psi, \varphi, x, y, z)$

$\mathrm{Det}(f') = -\rho^2 \operatorname{sen} \psi \neq 0 \qquad \rho \neq 0 \qquad \psi \neq k\pi \qquad k \in \mathbb{Z}.$

$U = \begin{cases} \rho \geq 0 \\ \psi \in [0, \pi] \\ \varphi \in [0, 2\pi) \end{cases}$ $\qquad f|_U$ es inyectiva y tiene inversa global.

$$f(x,y) = \begin{pmatrix} -3x+y^3 \\ -3y+x^3 \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix}$$

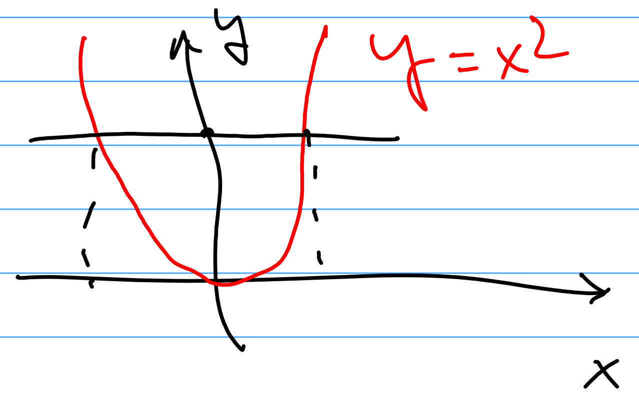
$f: \mathbb{R}^2 \to \mathbb{R}^2$ (x, y, u, v)

$f \in C^\infty(\mathbb{R}^2)$

$A(1,1,-2,-2)$ $B(1,2,5,-5)$

$$Df' = f' = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix} = \begin{pmatrix} -3 & 3y^2 \\ 3x^2 & -3 \end{pmatrix}$$

$\det f' = 9 - 9x^2y^2 = 9(1-x^2y^2) \neq 0$



en A T.F.Inv. no es aplicable.

$xy \neq 1$ $xy \neq -1$ es localmente invertible.

Sea $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$, $f(x, y) = (xe^y, xe^{-y})$.

1. ¿En qué puntos de $\mathbb{R}^2$ se puede garantizar la existencia de inversa local?
2. ¿Es f inyectiva? En caso negativo, encuentra el mayor subconjunto de $\mathbb{R}^2$ donde lo es, caracteriza su imagen y da una expresión de f^{-1}.

$$f(x,y) = \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} xe^{y} \\ xe^{-y} \end{pmatrix} \qquad Df = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix} = \begin{pmatrix} e^{y} & xe^{y} \\ e^{-y} & -xe^{-y} \end{pmatrix}$$

$$\det Df = -2x \neq 0 \qquad \forall (x,y) \quad x \neq 0$$

$$\left.\begin{aligned} x_1 e^{y_1} &= x_2 e^{y_2} \\ x_1 e^{-y_1} &= x_2 e^{-y_2} \end{aligned}\right\} \Rightarrow \begin{aligned} x_1 &= x_2 \\ y_1 &= y_2 \end{aligned} \qquad f(0,y) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$x = 0$

Debes: $(Df)^{-1}$ comparar con la dif. de la inversa $f^{-1}(u,v) = \begin{pmatrix} x \\ y \end{pmatrix}$

$$u = xe^{y} \qquad x \neq 0 \Rightarrow u \neq 0 \text{ y } v \neq 0$$

$$v = xe^{-y} \qquad uv = x^2 > 0 \qquad x = \pm\sqrt{uv} \quad \begin{cases} x > 0 \\ x < 0 \end{cases}$$

$$\frac{u}{v} = e^{2y} \qquad 2y = \log\frac{u}{v} \qquad y = \frac{1}{2}\log u/v$$

Sea $f : \mathbb{R}^2 \to \mathbb{R}^2$ definida por $f(x,y) = (\cosh x \cos y, \operatorname{senh} x \operatorname{sen} y)$.

a) Estudiar donde es f localmente invertible. ¿Lo es en los puntos $(0, n\pi)$, $n \in \mathbb{Z}$?

b) Si $S = \{(x,y) : x > 0\}$, ¿es f inyectiva en S?

c) Sea $U = \{(x,y) : x > 0,\ 0 < y < 2\pi\}$. Calcular $f(U)$ y ver que $f_{|U}$ es biyectiva sobre su imagen.

$$f(x,y) = \begin{pmatrix} \operatorname{ch} x \cos y \\ \operatorname{sh} x \operatorname{sen} y \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix}, \quad Df = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix} = \begin{pmatrix} \operatorname{sh} x \cos y & -\operatorname{ch} x \operatorname{sen} y \\ \operatorname{ch} x \operatorname{sen} y & \operatorname{sh} x \cos y \end{pmatrix}$$

$$\det Df = \underset{\substack{= \\ 0 \\ x=0}}{\operatorname{sh}^2 x}\ \underset{\substack{= \\ 1}}{\cos^2 y} + \underset{\substack{\neq \\ 0}}{\operatorname{ch}^2 x}\ \underset{\substack{= \\ 0}}{\operatorname{sen}^2 y} \overset{?}{\neq} 0 \qquad y = k\pi \qquad k \in \mathbb{Z}.$$

$$\det Df = 0 \qquad \forall\ (0, k\pi) \qquad k \in \mathbb{Z}.$$

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \cos k\pi \\ 0 \end{pmatrix} = \begin{pmatrix} (-1)^k \\ 0 \end{pmatrix} \begin{matrix} \nearrow \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \searrow \begin{pmatrix} -1 \\ 0 \end{pmatrix} \end{matrix}$$

$x > 0$. $\qquad x > 0$

$$\left\{ \begin{array}{l} \operatorname{ch} x \cos y = \operatorname{ch} x \cos(y + 2\pi k) \\ \operatorname{sh} x \operatorname{sen} y = \operatorname{sh} x \operatorname{sen}(y + 2\pi k) \\ y \in (0, 2\pi) \end{array} \right.$$

$$\begin{cases} u = \operatorname{ch} x \cos y \\ v = \operatorname{sh} x \operatorname{sen} y \end{cases}$$

2°) $\qquad Df^{-1} \qquad D^2 f^{-1}$

$$f^{-1}(u,v) \approx f^{-1}(a,b) + Df^{-1}(a,b)\begin{pmatrix} u-a \\ v-b \end{pmatrix} + \frac{1}{2} D^2 f^{-1}(a,b)\begin{pmatrix} u-a \\ v-b \end{pmatrix}$$