2.1) i) $F(x,y) = x^2+y^2-1=0$ $\quad y(x)$ en entorno de $A(\sqrt{2}/2, -\sqrt{2}/2)$

$F \in C^\infty(\mathbb{R}^2)$ $\quad F'_y = 2y\Big|_A \neq 0$ $\quad F(\sqrt{2}/2, -\sqrt{2}/2) = 0$

T.F.I $\Rightarrow$ $\exists\, y(x)$ en el entorno de A t.q. $F(x,y(x))=0$.

$$y' = \frac{\partial y}{\partial x} = -\frac{F'_x}{F'_y} = -\frac{2x}{2y}\Big|_A = 1$$

ii) $F = x^3 \log x^2 + y^2 = 0$ en $(1,0)$ $\quad$ ¿$x(y)$?

$F(1,0)=0$ $\quad F \in C^\infty(\mathbb{R}^2)$ $\quad F'_x \neq 0 = 3x^2 \log x^2 + x^3 \cdot \dfrac{2x}{x^2}\Big|_{(1,0)} = 2 \neq 0$

Se cumplen las hip. del T.F.I $\Rightarrow$ $\exists\, x(y)$ en entorno U de $(1,0)$

$$x'(y) = -\frac{F'_y}{F'_x} = -\frac{2y}{3x^2\log x^2 + 2x^2}\Big|_{(1,0)} = 0 \qquad x'(0)=0 \;!!$$

$x(y) \in C^\infty(U)$

$y=0$ candidato a extremo!

$x(y)$ tiene extremo en $y=0$!! $\qquad x'(0)=0$!! $\qquad x''(0)$??

$F(x(y),y) = x^3 \log x^2 + y^2 = 0$

$$F_y = 3x^2 x' \log x^2 + x^3 \frac{2xx'}{x^2} + 2y = 0$$

$$3x^2 \log x^2 \cdot x' + 2x^2 x' + 2y = 0$$

$$F_y = (3x^2 \log x^2 + 2x^2)\, x' + 2y = 0$$

$$F_{yy} = 0 = \left(6xx' \log x^2 + 3x^2 \frac{1}{x^2} 2x' + 4xx'\right) x' + (3x^2 \log x^2 + 2x^2)\, x'' + 2$$

$x=1,\ y=0,\ x'=0 \qquad 2x''+2=0 \qquad x''(0)=-1<0$

$x(y) \in C^\infty(U) \qquad x'(0)=0 \quad x''(0)<0 \Rightarrow$ máximo local en $y=0$.

$F(x,y,z) = x^2 y + e^x + z = 0$ $\qquad x(y,z) \quad$ t.q. $\quad x(y,z) = 0$

$A(0,y,-1) \quad \forall y \in \mathbb{R} \qquad F(0,y,z) = z+1 = 0 \quad \Rightarrow z = -1$

$F'_x = 2xy + e^x \Big|_{(0,y,-1)} = 1 \neq 0 \qquad F \in C^\infty(\mathbb{R}^3) \quad \text{TFI} \Rightarrow \exists\, x(y,z)$ en U

(U entorno de $(0,y,-1)$ $\forall y \in \mathbb{R}$) t.q.

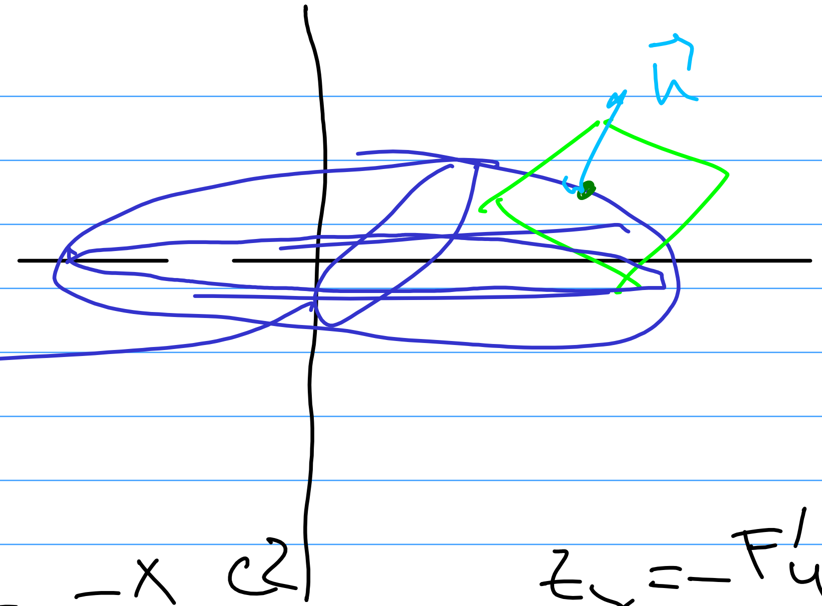
$F(x,y,z) = 0 \quad x$ es una función implícita $\Rightarrow \quad x \in C^\infty(U)$

$$\frac{\partial F}{\partial y} = 2x x_y \, y \overbrace{+}^{+x^2} e^x x_y \Big|_A = 0 \quad \Rightarrow \quad x_y = 0$$

$x(y,z)$

$$\frac{\partial F}{\partial z} = 2x x_z y + e^x x_z + 1 \Big|_A = 0 \quad \Rightarrow \quad x_z = -1$$

$$F = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0 \qquad z = z(x,y)$$



$$F'_z = \frac{2z}{c^2} \neq 0 \Rightarrow z \neq 0 \qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\vec{n} = (z_x, z_y, -1) \qquad z_x = -\frac{F'_x}{F'_z} = -\frac{\cancel{2}x c^2}{a^2 \cancel{2} z} = -\frac{x}{z}\frac{c^2}{a^2}, \quad z_y = -\frac{F'_y}{F'_z} = -\frac{y c^2}{z b^2}$$

(x_0, y_0, z_0) $\quad z_0 \neq 0$

$$\frac{x_0 c^2}{a^2 z_0}(x - x_0) + \frac{y_0 c^2}{z_0 b^2}(y - y_0) + (z - z_0) = 0$$

$$\frac{x_0}{a^2}(x - x_0) + \frac{y_0}{b^2}(y - y_0) + \frac{z_0}{c^2}(z - z_0) = 0 \qquad z_0 \neq 0$$

$\underline{y(x,z)}$? $\quad F'_y = \frac{2y}{b^2} \neq 0 \quad y \neq 0 \quad$ TFI $\Rightarrow \exists\, y(x,z)$. dif...

$$y_x = -\frac{F'_x}{F'_y} = -\frac{\cancel{2}x}{a^2 \cancel{2} y} b^2, \quad y_z = -\frac{F'_z}{F'_y} = -\frac{2z}{c^2}\frac{b^2}{2y} = - \qquad y \neq 0$$

$$\vec{n} = (y_x, y_z, -1) = \left(+ \frac{x_0 b^2}{y_0 a^2}, + \frac{z_0 b^2}{y_0 c^2}, +1 \right) \qquad (x-x_0, z-z_0, y-y_0)$$

$$\frac{x_0 b^2}{a^2 y_0}(x-x_0) + \frac{z_0 b^2}{y_0 c^2}(z-z_0) + (y-y_0) = 0 \qquad y_0 \neq 0$$

$$\frac{x_0}{a^2}(x-x_0) + \frac{z_0}{c^2}(z-z_0) + \frac{y_0}{b^2}(y-y_0) = 0$$

Debemos $\Rightarrow$ $x(y,z)$ $\Rightarrow$ comprobad $\qquad x_0 \neq 0$