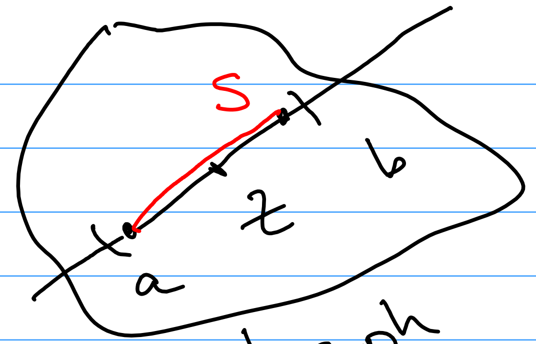

$A \in \mathbb{R}^n$

$z \in S$

$U \in A$

$\langle v, f(b) - f(a) \rangle = \langle v, Df(z)(b-a) \rangle$

$U = \{ x = a + (b-a)t \ , \ t \in (-\delta, 1+\delta) \}$ $\quad \delta$ chico $\quad \underline{\underline{U \subset A}}$

f es diferenciable en U,

$f: \mathbb{R}^n \to \mathbb{R}^m$, $\quad v \in \mathbb{R}^m$

$F: \mathbb{R} \to \mathbb{R}$

$$F(t) = \langle v, f(a+(b-a)t) \rangle = \sum_{k=1}^{m} v_k f_k(a+(b-a)t)$$

f es dif. en A $\quad$ y $\; g: a+(b-a)t \quad g: \mathbb{R} \to \mathbb{R}^n$ es dif. en $(-\delta, 1+\delta)$

$F = f \circ g$ es dif en $(-\delta, 1+\delta)$ es cont. $[0,1]$ y dif $(0,1)$

$$F(1) - F(0) = F'(\xi)(1-0) = F'(\xi) \qquad \xi \in (0,1)$$

$F(1) = \langle v, f(b) \rangle \quad F(0) = \langle v, f(a) \rangle \qquad F(1) - F(0) = \langle v, f(b) - f(a) \rangle$

$$F'(\xi) = \sum_{k=1}^{m} v_k \frac{\partial}{\partial t} f_k(a+(b-a)t) \qquad \frac{\partial x_j}{\partial t} = \frac{\partial}{\partial t}(a_j + (b_j - a_j)t) = b_j - a_j$$

$$\frac{\partial f_k(a+(b-a)t)}{\partial t} = \sum_{j=1}^{n} \frac{\partial f_k}{\partial x_j}(a+(b-a)t)\frac{\partial x_j}{\partial t} = \sum_{j=1}^{n} \frac{\partial f_k}{\partial x_j}(a+(b-a)t)(b_j-a_j) =$$

$$= Df_k(a+(b-a)t)(b-a)$$

$\xi \in (0,1) \Rightarrow a+(b-a)\xi \in S$

$$F'(\xi) = \sum_{k=1}^{m} v_k \cdot Df_k(a+(b-a)\xi)(b-a) = \langle v, Df(a+(b-a)\cdot\xi)(b-a)\rangle$$

$$\Rightarrow \quad \langle v, f(b)-f(a)\rangle = \langle v, Df(z)(b-a)\rangle \qquad z \in \mathring{S}$$

$f: \begin{pmatrix} f_1 \\ \vdots \\ f_m \end{pmatrix}$ $\qquad \cancel{v}\cdot(f_k(b)-f_k(a)) = \cancel{v}\cdot Df_k(z_k)(b-a)$ $\qquad z_k \in$ interior de S.

$v = e_1, \ldots e_m \qquad \forall e_k \quad \exists z_k.$

T.V.M $\qquad \langle v, f(b)-f(a)\rangle = \langle v, Df(z)(b-a)\rangle \qquad z \in$ int. de S

$v = f(b)-f(a)$

$$\langle f(b)-f(a),\, f(b)-f(a)\rangle = \|f(b)-f(a)\|^2 = \langle f(b)-f(a),\, Df(z)(b-a)\rangle \leq$$

$$\leq \|f(b)-f(a)\|\,\|Df(z)(b-a)\|$$

Si $f(a)=f(b)$ lo anterior es trivial. $f(b)\neq f(a) \Rightarrow$

$$\underline{\|f(b)-f(a)\|} \leq \|Df(z)(b-a)\| \leq \underline{\|Df(z)\|\,\|b-a\|} = M\,\|b-a\|$$

* $Df(z)$ $\forall z$ es aplic. lineal def. sobre $\mathbb{R}^n$ $\Rightarrow$ $Df(z)$ es acotada. $\Rightarrow$!!!

$Df(z)=0 \quad \forall\, z\in A \Rightarrow f(x)$ es const

$\forall a,b\in A \quad \langle v,\, f(b)-f(a)\rangle = \langle v,\, \cancel{Df(z)}(b-a)\rangle = 0 \qquad \forall v, \; v=f(b)-f(a)$

$$\langle f(b)-f(a),\, f(b)-f(a)\rangle = 0 \Rightarrow \|f(b)-f(a)\|^2=0 \Rightarrow f(a)=f(b)$$

$\forall a,b\in A$.