$$f(x,y) = e^{ax+by} \qquad P_2(x,y,0,0) \qquad h = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$P_2(x,y) = f(0,0) + Df(0,0)\begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2} D^2 f(0,0)\begin{pmatrix} x \\ y \end{pmatrix}$$

$$Df(0,0) = \left(\frac{\partial f}{\partial x} \quad \frac{\partial f}{\partial y}\right)\Bigg|_{(0,0)} = (a \quad b) \qquad f \in C^2(\mathbb{R}^2)$$

$$D^2 f(0,0) \rightarrow H_f(0,0) = \begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{pmatrix} = \begin{pmatrix} a^2 & ab \\ ab & b^2 \end{pmatrix}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(a e^{ax+by}\right) = a^2 e^{ax+by}\Big|_{(0,0)}$$

$$P_2(x,y) = 1 + (a\ b)\begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2}(x\ y)\begin{pmatrix} a^2 & ab \\ ab & b^2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$= 1 + (ax+by) + \frac{1}{2}(a^2x^2 + b^2y^2 + 2abxy)$$

$$D^k f(0,0)(h_x,h_y) = \left(h_x\frac{\partial}{\partial x} + h_y\frac{\partial}{\partial y}\right)^k e^{ax+by}\Big|_{(x,y)=(0,0)}$$

$$h = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$f = e^{ax+by} \to e^z = 1 + z + \frac{z^2}{2} + \dots + \frac{z^{17}}{17!} + \dots + \frac{z^n}{n!} + o(\|h\|^n)$$

$$= 1 + (ax+by) + \frac{(ax+by)^2}{2} + \dots + \frac{(ax+by)^n}{n!}$$

1.40. $\sin xy$ $\qquad \sin z = z - z^3/6 + o(\|z^3\|)$

$f(x,y) = \sin xy = \underbrace{xy}_{P_2(x,y)} - \frac{(xy)^3}{6} + \dots \qquad f(0,0) = 0$

$\frac{\partial f}{\partial x} = y \cos xy \qquad \frac{\partial f}{\partial y} = x \cos xy \qquad Df(0,0) = (0 \quad 0)$

$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}(y \cos xy) = y^2(-\sin xy)\Big|_{00} = 0$

$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}(x \cos xy) = \underset{\overset{\|}{1}}{\cos xy} + \cancel{xy(-\sin xy)}\Big|_{00} = 1$

$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}(x \cos xy) = -x^2 \sin xy\Big|_{(00)} = 0$

$$P_2(x,y) = 0 + \begin{pmatrix} 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2}\begin{pmatrix} x & y \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} =$$

$$= \underline{\underline{xy}}$$

$\underline{1.40}$ $f(x,y,z) = e^{x^2+y^2+z^2}$ $(0,0,0)$

$$\frac{\partial f}{\partial x} = 2x\, f(x,y,z) = 2x e^{x^2+y^2+z^2}$$

$$\frac{\partial f}{\partial y} = 2y\, e^{x^2+y^2+z^2} \qquad \frac{\partial f}{\partial z} = 2z e^{x^2+y^2+z^2}$$

$$\frac{\partial^2 f}{\partial x^2} = 2e^{x^2+y^2+z^2} + 4x^2 e^{x^2+y^2+z^2} = 2e^{x^2+y^2+z^2}(2x^2+1)$$

$$\frac{\partial^2 f}{\partial y^2} = 2e^{x^2+y^2+z^2}(2y^2+1) \qquad \frac{\partial^2 f}{\partial z^2} = 2e^{x^2+y^2+z^2}(2z^2+1)$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left[2ye^{x^2+y^2+z^2}\right] = 4xye^{x^2+y^2+z^2} = \frac{\partial^2 f}{\partial y \partial x}$$

$$\frac{\partial^2 f}{\partial x \partial z} = \frac{\partial}{\partial x}\left[2ze^{x^2+y^2+z^2}\right] = 4xze^{x^2+y^2+z^2}$$

$$\frac{\partial^2 f}{\partial y \partial z} = \frac{\partial}{\partial y}\left[2ze^{x^2+y^2+z^2}\right] = 4yze^{x^2+y^2+z^2}$$

1) $(0,0,0)$ $\quad P_2(x,z,z) = f(0,0,0) + Df(0,0,0)\begin{pmatrix} x \\ y \\ z \end{pmatrix} + \frac{1}{2}(x\ y\ z)\, H_f(0,0,0)\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$$= 1 + \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} + \frac{1}{2}(x\ y\ z)\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$= 1 + \frac{1}{2}\left(2x^2+2y^2+2z^2\right) = 1+x^2+y^2+z^2$$

$$e^z = 1 + z + \cdots \qquad z = x^2+y^2+z^2$$

2) (a,b,c)

$$P_2(x,y,z) = e^{a^2+b^2+c^2}\left[1 + \begin{pmatrix} 2a & 2b & 2c \end{pmatrix}\begin{pmatrix} x-a \\ y-b \\ z-c \end{pmatrix} + \right.$$

$$\frac{1}{2}\begin{pmatrix} x-a & y-b & z-c \end{pmatrix}\begin{pmatrix} 2(2a^2+1) & 4ab & 4ac \\ 4ab & 2(2b^2+1) & 4bc \\ 4ac & 4bc & 2(2c^2+1) \end{pmatrix}\begin{pmatrix} x-a \\ y-b \\ z-c \end{pmatrix} =$$

$$= e^{a^2+b^2+c^2}\left[1 + 2\begin{pmatrix} a & b & c \end{pmatrix}\begin{pmatrix} x-a \\ y-b \\ z-c \end{pmatrix} + \begin{pmatrix} x-a & y-b & z-c \end{pmatrix}\begin{pmatrix} 2a^2+1 & 2ab & 2ac \\ 2ab & 2b^2+1 & 2bc \\ 2ac & 2bc & 2c^2+1 \end{pmatrix}\begin{pmatrix} x-a \\ y-b \\ z-c \end{pmatrix}\right.$$

TERMINARLO!!

1.41. $f(x,y) = x \operatorname{sen} y + y \operatorname{sen} x$ orden 8 en $(0,0)$

$\operatorname{sen} y = y - y^3/6 + y^5/5! - \frac{y^7}{7!} + \dots$ $\operatorname{sen} x = x - x^3/6 + x^5/5! - \frac{x^7}{7!} \dots$

$P_8(x,y) = 2xy - \frac{xy^3}{6} + \frac{xy^5}{5!} - \frac{xy^7}{7!} + \cancel{yx} - yx^3/6 + yx^5/5! - yx^7/7!$

Ejercicio: $D^4 f(0,0)$.

$f(x,y)$ $P_2(x,y)$ en $(1,2)$ $f(x,y) = x^3 + y^2 + xy^2$

$$P_2(x,y) = f(1,2) + Df(1,2)\begin{pmatrix} x-1 \\ y-2 \end{pmatrix} + (x-1 \;\; y-2)\, H_f(1,2)\begin{pmatrix} x-1 \\ y-2 \end{pmatrix}$$

$\quad \overset{\shortparallel}{9}$

$$Df(x,y) = \Big(\underbrace{3x^2+y^2}_{f_x},\ \underbrace{2y+2xy}_{f_y}\Big)\Big|_{(1,2)} = (7 \quad 8)$$

$$H_f = \begin{pmatrix} 6x & 2y \\ 2y & 2x+2 \end{pmatrix}\Bigg|_{(1,2)} = \begin{pmatrix} 6 & 4 \\ 4 & 4 \end{pmatrix}$$

$$P_2(x,y) = 9 + \begin{pmatrix} 7 & 8 \end{pmatrix}\begin{pmatrix} x-1 \\ y-2 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} x-1 & y-2 \end{pmatrix}\begin{pmatrix} 6 & 4 \\ 4 & 4 \end{pmatrix}\begin{pmatrix} x-1 \\ y-2 \end{pmatrix}$$

$$= 9 + 7(x-1) + 8(y-2) + 3(x-1)^2 + 2(y-2)^2 + 4(x-1)(y-2)$$

$$= 9 + 7x - 7 + \cancel{8y} - \cancel{16} + 3x^2 - 6x + 3 + 2y^2 - \cancel{8y} + \cancel{8} + 4xy - 8x - 4y + \cancel{8}$$

$$= 5 - 7x - 4y + 3x^2 + 2y^2 + 4xy$$