$$f(x,y)=\begin{cases}\dfrac{|x|^\alpha \arctan y}{\sqrt{x^2+y^2}}, & \text{si } (x,y)\neq(0,0),\\ 0, & \text{si } (x,y)=(0,0).\end{cases}$$

$\alpha \geq 0$

$$\lim_{(x,y)\to 0} \frac{|x|^\alpha \operatorname{arctg} y}{\sqrt{x^2+y^2}} = ?$$

$\operatorname{arctg} z \sim z$

$y = mx$

$$\frac{|x|^\alpha\, |y|}{\sqrt{x^2+y^2}} \leq \begin{cases} \dfrac{|x|^\alpha \cancel{|y|}}{\cancel{|y|}} \to 0 & \alpha > 0 \\ \dfrac{|x|^\alpha |y|}{|x|} \to 0 & \alpha \geq 1 \\ \dfrac{|x||y|}{\sqrt{x^2+y^2}} \quad |x|^{\alpha-1/2}|y|^{1/2} \to 0 & \alpha \geq 1/2 \end{cases}$$

$\leq 1/2$

$$\frac{|x|^\alpha |m| \cancel{|x|}}{\cancel{|x|}\sqrt{1+m^2}} \to 0 \qquad \alpha > 0$$

$\Downarrow$

$$\forall \alpha > 0 \quad \lim_{(x,y)\to 0} f(x,y) = 0$$

$\underline{\underline{\alpha > 0}}$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{x\to 0} \frac{\cancel{f(x,0)}^{0} - \cancel{f(0,0)}^{0}}{x} = 0$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{y\to 0} \frac{\cancel{f(0,y)}^{0} - \cancel{f(0,0)}^{0}}{y} = 0$$

$\underline{\underline{\alpha > 0}}$? f es dif. en (0,0) ?

$$\frac{f(x,y) - \cancel{f(0,0)}^{0} - \cancel{Df(0,0)\binom{x}{y}}^{0}}{\sqrt{x^2+y^2}} \overset{?}{\longrightarrow} 0$$

$\alpha > 1$

$$\left| \frac{|x|^{\alpha} \arctan y}{x^2+y^2} \right| \sim \frac{|x|^{\alpha}|y|}{x^2+y^2} = \underbrace{\frac{|x||y|}{x^2+y^2}}_{\leq 1/2} |x|^{\alpha-1} \to 0$$

¿ $\alpha = 1$?

$$\frac{|x| \operatorname{arctg} y}{x^2+y^2} \not\to 0 \qquad y = mx \qquad \frac{|x| \operatorname{arctg} mx}{x^2(1+m^2)} \xrightarrow[x \to 0]{} \frac{|x|}{x} \frac{m}{1+m^2}$$

$x > 0$

$\alpha > 1$ No es. dif. en $(0,0)$

Sea la función $g : \mathbb{R}^3 \mapsto \mathbb{R}$, $g(x, y, z) = x^2 + ye^{x\,z}$.

¿Cuánto vale la derivada g en el punto $(1, 2, 0)$ según la dirección del vector $(3/4, \sqrt{3}/2, 1/2)$.

$$\frac{\partial g}{\partial x} = 2x + yze^{xz} \qquad \frac{\partial g}{\partial y} = e^{xz} \qquad \frac{\partial g}{\partial z} = xye^{xz}$$

$$Dg(x,y,z) = \left(2x + yze^{xz},\ e^{xz},\ xye^{xz}\right) = \left(\frac{\partial g}{\partial x}\ \frac{\partial g}{\partial y}\ \frac{\partial g}{\partial z}\right)$$

$$\frac{9}{16} + \frac{3}{4} + \frac{1}{4} = \frac{9+12+4}{16} = \frac{25}{16}$$

$$\|u\| = \frac{5}{4}$$

$$D_u g(1,2,0) = Dg(1,2,0)(u)$$

$$= \frac{4}{5}\begin{pmatrix} 2 & 1 & 2 \end{pmatrix}\begin{pmatrix} 3/4 \\ \sqrt{3}/2 \\ 1/2 \end{pmatrix} =$$

$$\frac{4}{5}\left(\frac{8}{4} + \frac{\sqrt{3}}{2} + 1\right) = \frac{\cancel{4}}{5}\left(\frac{8 + 2\sqrt{3} + 4}{\cancel{4}}\right) = \frac{12 + 2\sqrt{3}}{5}$$

Plano tang. en $(1,2,0)$ $\qquad \vec{u} = (2 \;,\; 1 \;,\; 2 \;\; -1)$

$\langle \vec{u} \,,\, x-1 \;\; y-2 \;\; z \;\; w-3 \rangle = 0$

$g(x,y,z) = w$

$w_0 = g(1,2,0) = 3$

$2(x-1) + (y-2) + 2z - (w-3) = 0$ $\qquad 2x + y + 2z - w - 1 = 0$

$\dfrac{\partial g}{\partial x} = 2x + yz e^{xz}$ $\qquad \dfrac{\partial g}{\partial y} = e^{xz}$ $\qquad \dfrac{\partial g}{\partial z} = xy e^{xz}$ $\quad (A(1,2,0)$

$\dfrac{\partial^2 g}{\partial x^2} = 2 + yz^2 \cdot e^{xz} \Big|_A = 2$ $\qquad \dfrac{\partial^2 g}{\partial y^2} = 0$ $\qquad \dfrac{\partial^2 g}{\partial z^2} = x^2 y e^{xz} = 2$

$\dfrac{\partial^2 g}{\partial x \partial y} = z e^{xz} \Big|_A = 0$ $\qquad \dfrac{\partial^2 g}{\partial x \partial z} = y e^{xz} + xyz e^{xz} \Big|_A = 2$ $\qquad \dfrac{\partial^2 g}{\partial y \partial z} = x e^{xz} \Big|_A = 1$

$A = (1,2,0)$

$$P_2(x,y) = g(A) + Dg(A)\begin{pmatrix} x-1 \\ y-2 \\ z \end{pmatrix} + \frac{1}{2} D^2 g(A)\begin{pmatrix} x-1 \\ y-2 \\ z \end{pmatrix} =$$

$$= 3 + \begin{pmatrix} 2 & 1 & 2 \end{pmatrix}\begin{pmatrix} x-1 \\ y-2 \\ z \end{pmatrix} + \frac{1}{2}\begin{pmatrix} x-1 & y-2 & z \end{pmatrix}\begin{pmatrix} 2 & 0 & 2 \\ 0 & 0 & 1 \\ 2 & 1 & 2 \end{pmatrix}\begin{pmatrix} x-1 \\ y-2 \\ z \end{pmatrix}$$

$Dg(A)(h)$ $\quad h$

$$= 3 + \boxed{2(x-1) + (y-2) + 2z} + (x-1)^2 + z^2 + 2(x-1)z + z(y-2)$$

$$h = \begin{pmatrix} x-1 \\ y-2 \\ z \end{pmatrix}$$

$$D^3 g(1,2,0) \qquad \left(h_x \frac{\partial}{\partial x} + h_y \frac{\partial}{\partial y} + h_z \frac{\partial}{\partial z}\right)(x^2 + y e^{xz})$$

g $\qquad x,y,z \to (1,2,0)$

$$h_x \frac{\partial g}{\partial x} + h_y \frac{\partial g}{\partial y} + h_z \frac{\partial g}{\partial z} \Bigg| \quad (x,y,z) \to (1,2,0)$$

$$\frac{\partial g(A)}{\partial x}(x-1) + \frac{\partial g(A)}{\partial y}(y-2) + \frac{\partial g(A)}{\partial z} z \quad \Big|$$

$$\underbrace{\left(h_x \frac{\partial}{\partial x} + h_y \frac{\partial}{\partial y} + h_z \frac{\partial}{\partial z} \right)}_{L} \Big(\quad L \quad \Big) g$$

$$\left(h_x \frac{\partial}{\partial x} + h_y \frac{\partial}{\partial y} + h_z \frac{\partial}{\partial z} \right) \left(h_x \frac{\partial g}{\partial x} + \cdot + h_z \frac{\partial g}{\partial z} \right) \Big|$$

$x, y, z = (1, 2, 0)$

$y \to$

$$\sum_{i=1\, j=1\, k=1}^{3} \frac{\partial^3}{\partial x_i \partial x_j \partial x_k} g(A) (x_i - a_i)(x_j - a_j)(x_k - a_k)$$

$x_1 = x \quad x_2 = y \quad x_3 = z$

$a_1 = 1 \quad a_2 = 2 \quad a_3 = 0$

Sea $f:\mathbb{R}^2\setminus\{(0,0)\}\mapsto\mathbb{R}$, $f(x,y)=\dfrac{|x|^\alpha e^{3x^2}\arcsin(2x)}{\sqrt{2x^2+y^2}}$, $\alpha\geq 0$.

$(0,0)$!!

$f(0,0)=0$ $\quad \alpha>0$

$\lim_{(x,y)\to 0} f(x,y)=0$

$z\to 0 \quad e^z\sim 1+z$

$\cancel{e^{3x^2}\sim 3x^2}$

$|f(x,y)|\sim\dfrac{|x|^\alpha\cdot 1\cdot|2x|}{\sqrt{2x^2+y^2}}=2\dfrac{|x|^{\alpha+1}}{\sqrt{2x^2+\cancel{y^2}}}\leq 2\dfrac{|x|^{\alpha+1}}{\sqrt{2}\,|x|}$?!

$=\sqrt{2}\,|x|^\alpha\longrightarrow 0$ si $\alpha>0$

$\alpha=0$ $\quad\dfrac{e^{3x^2}\operatorname{arsen}2x}{\sqrt{2x^2+y^2}}\to$ N.T.L. $\quad y=mx \quad x>0$

$\dfrac{e^{3x^2}\operatorname{arsen}2x}{|x|\sqrt{2+m^2}}\sim\dfrac{1\cdot 2x}{|x|\sqrt{2+m^2}}$

$\alpha > 0 \quad \frac{\partial f}{\partial x}(0,0) = \lim_{x\to 0} \frac{f(x,0) - \cancel{f(0,0)}^{0}}{x} = \lim_{x\to 0} \frac{|x|^\alpha e^{3x^2} \cancel{\arcsin} \, 2\cancel{x}}{\sqrt{2}\,\cancel{x}\, x} = \lim_{x\to 0} \sqrt{2}|x|^{\alpha-1} = \begin{cases} 0 & \alpha > 1 \\ \sqrt{2} & \alpha = 1 \end{cases} \quad \alpha \geq 1$

$\alpha > 1. \quad Df(0,0) = (0 \quad 0) \quad \left\{ \frac{\partial f}{\partial y}(0,0) = 0 \right.$

Diferenciabilidad

$\underline{\underline{\alpha > 1}} \qquad \frac{f(x,y) - \cancel{f(0,0)}^{0} - \cancel{Df(0,0)}^{0}\begin{pmatrix} x \\ y \end{pmatrix}}{\sqrt{x^2+y^2}} = \frac{|x|^\alpha e^{3x^2} \arcsin 2x}{\sqrt{2x^2+y^2}\,\sqrt{x^2+y^2}}$

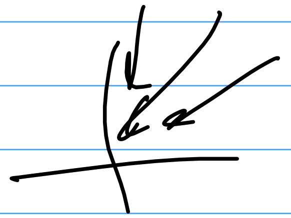
$\frac{|x|^\alpha \cdot 2|x|}{\sqrt{2x^2+y^2}\,\sqrt{x^2+y^2}} \leq \frac{2|x|^{\alpha+1}}{x^2+\cancel{y^2}} \leq 2|x|^{\alpha-1} \longrightarrow 0 \qquad \alpha > 1$

$\alpha = 1 \qquad Df(0,0) = (\sqrt{2} \quad 0)$

$\alpha = 1 \,! \qquad \frac{\left[\frac{|x| \cancel{e^{3x^2}}^{1} \arcsin 2x}{\sqrt{2x^2+y^2}} - \sqrt{2}\,x \right]}{\sqrt{x^2+y^2}} \longrightarrow 0$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{y \to 0} \frac{\cancel{f(0,y)}^{0} - \cancel{f(0,0)}^{0}}{y} = 0 \qquad \forall \alpha > 0$$

$\alpha = 1 \qquad$ N.T.C ≥ 0



$y = mx$

$\underline{\underline{x \to 0}}$

$$\frac{\dfrac{\cancel{|x|}\, 2x}{\cancel{|x|}\sqrt{2+m^2}} - \sqrt{2}\,x}{|x|\sqrt{1+m^2}} = \left(\frac{2}{\sqrt{2+m^2}} - \sqrt{2}\right) \Big/ \sqrt{1+m^2}$$

1) $$\lim_{(x,y)\to(0,0)} \frac{x^2+3y^2+6xy-5xy^2}{\sqrt{x^2+2y^2}} = 0$$

2) $$\lim_{(x,y)\to(0,0)} \frac{\sin(x|y|^{3/2})}{|x|^2+|y|} = 0$$

1) $$\left| \frac{x^2+3y^2}{\sqrt{x^2+2y^2}} + \frac{xy(6-5y)}{\sqrt{x^2+2y^2}} \right| \le \frac{x^2+3y^2}{\sqrt{x^2+2y^2}} + \frac{|xy|(6+5|y|)}{\sqrt{x^2+2y^2}}$$

$$\le \frac{\overset{2x^2+x^2}{3x^2}+3y^2}{\sqrt{x^2+\cancel{2}y^2}} \to 0 \quad + \quad \frac{|xy|^{1/2}}{\sqrt{x^2+\cancel{2}y^2}}\; |xy|^{1/2}(6+5|y|)$$

$$3\sqrt{x^2+y^2} \qquad\qquad \le \sqrt{1/2} \qquad\qquad \downarrow 0$$

2) $$\sim \frac{|x||y|^{3/2}}{|x|^2+|y|} = \frac{|x||y|^{1/2}}{|x|^2+|y|}\;|y|$$

$$\le 1/2 \qquad\qquad |y| \downarrow 0$$

Sea $g : \mathbb{R}^2 \mapsto \mathbb{R}$, $g(x,y) = xe^y + ye^x + 2xy$.

1. Decide si g es diferenciable en $\mathbb{R}^2$. Justifica la respuesta.
2. Calcula las derivadas parciales de orden uno y dos.
3. ¿Cuánto vale la derivada g en el punto $(0,0)$ según la dirección del vector $(2,1)$.
4. Escribe el polinomio de Taylor de orden 2 en el punto $(0,0)$.

$$\frac{\partial g}{\partial x} = e^y + ye^x + 2y \qquad \frac{\partial g}{\partial y} = xe^y + e^x + 2x$$

$$\frac{\partial^2 g}{\partial x^2} = ye^x \qquad \frac{\partial^2 g}{\partial y^2} = xe^y \qquad \frac{\partial^2 g}{\partial x \partial y} = e^y + e^x + 2$$

$$\left\{ \begin{array}{l} Dg(0,0) = (1 \quad 1) \\ u = (2,1)/\sqrt{5} \end{array} \right.$$

$$D_u g(0,0) = Dg(0,0)(u) = \frac{1}{\sqrt{5}}(1 \quad 1)\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

plano $\vec{n} = (1 \ 1 \ -1)$ $\quad \langle \vec{n}, (x \ y \ z) \rangle = 0$ $\quad x + y - z = 0$

$g = xe^y + ye^x + 2xy$ en $(0,0)$ $\quad e^z = 1 + z + \frac{z^2}{2!} \ldots$

$$P_2(x,y) = x(1+y) + y(1+x) + 2xy = x + xy + y + xy + 2xy$$

$$= x + y + 4xy$$

$$P_2(x,y) = g(0,0) + \overset{(1 \ 1)}{Dg(0,0)} \begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2} D^2 g(0,0) \begin{pmatrix} x \\ y \end{pmatrix} =$$

$$= x + y + \frac{1}{2} (x \ y) \begin{pmatrix} 0 & 4 \\ 4 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = x + y + 4xy$$