$\boxed{1.31}$ $f(x,y) = \dfrac{|\operatorname{sen} x|^\alpha \operatorname{artg}(xy)}{x\sqrt{x^2+y^2}}$ $\quad x \neq 0 \qquad \alpha > 0$

$f(0,a) = 0$

$\operatorname{artg} z \sim z$
$\operatorname{sen} x \sim x$

1º $\displaystyle\lim_{(x,y)\to(0,a)} f(x,y)$

$\dfrac{|x|^\alpha \cancel{x} a}{\cancel{x}\sqrt{x^2+a^2}} \xrightarrow{x\to 0} \begin{cases} 0 & a \neq 0 \\ & a = 0 \end{cases}$

$\left| \dfrac{|x|^\alpha \cancel{x} y}{\cancel{x}\sqrt{\cancel{x^2}+y^2}} \right| \leq \dfrac{|x|^\alpha |\cancel{y}|}{|\cancel{y}|} = |x|^\alpha \to 0 \quad \underline{\alpha > 0}$

(Debes $(0,a)$??)

$a > 0 \Rightarrow f$ es cont en $(0,0)$

$\dfrac{\partial f}{\partial x}(0,0) = \displaystyle\lim_{x\to 0} \dfrac{\overset{0}{f(x,0)} - \overset{0}{f(0,0)}}{x} = 0 \qquad \dfrac{\partial f}{\partial y}(0,0) = \displaystyle\lim_{y\to 0} \dfrac{\overset{0}{f(0,y)} - \overset{0}{f(0,0)}}{y} = 0$

$\dfrac{f(x,y) - \overset{0}{f(0,0)} - \begin{pmatrix} 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}}{\sqrt{x^2+y^2}} \overset{?}{\longrightarrow} 0$

$\alpha > 1$

$$\left| \frac{|\operatorname{sen} x|^\alpha \operatorname{artg}(xy)}{x\,(x^2+y^2)} \right| \sim \frac{|x|^\alpha\, |\cancel{x}|\,|y|}{|\cancel{x}|\,(x^2+y^2)} \leq \begin{cases} |x|^{\alpha-2}\,|y| & \alpha \geq 2 \\ \dfrac{|x|^\alpha}{|y|} \end{cases}$$

$$|x|^{\alpha-1} \frac{|x||y|}{x^2+y^2} \leq 1/2$$

$$\leq \frac{1}{2}|x|^{\alpha-1} \longrightarrow 0 \qquad \alpha > 1$$

¿$\alpha = 1$?

$$\frac{|\operatorname{sen} x| \operatorname{artg} xy}{x\,(x^2+y^2)} \sim \frac{m\cancel{x}\; m\cancel{x^2}}{\cancel{x}\cdot\cancel{x^2}(1+m^2)} = \frac{m^2}{1+m^2} \qquad y = mx$$

N.T.L.

Diferenciable $\forall \alpha > 1$

dif. ?? en $(0,a)$ $a \neq 0$

$\alpha = 1$ $\quad D_u f(0,0) = \lim_{\lambda \to 0} \dfrac{f(\lambda u_x, \lambda u_y) - \cancel{f(0,0)}^{0}}{\lambda} =$

$\|u\| = 1$

$$= \lim_{\lambda \to 0} \frac{|\operatorname{sen} \lambda u_x| \operatorname{arctg}(\lambda^2 u_x u_y)}{\lambda u_x \sqrt{\lambda^2 u_x^2 + \lambda^2 u_y^2}\ \lambda} \sim \frac{|\cancel{\lambda} u_x| \cancel{\lambda^2}\ u_x u_y}{\cancel{\lambda} |\cancel{\lambda}|\ \cancel{\lambda}} = |u_x| u_x u_y$$

$u \to D_u f(0,0)$ N. es lineal

$\boxed{1.28}$

$$f(x,y) = \begin{cases} \dfrac{1-\cos xy}{y^2} & y \neq 0 \\[2mm] \dfrac{x^2}{2} & y = 0 \end{cases} \qquad \underline{\underline{(a,0)}} \qquad a \in \mathbb{R}$$

$$\lim_{(x,y)\to(a,0)} \frac{1-\cos xy}{y^2} = \lim_{(x,y)\to(a,0)} \frac{(xy)^2/2}{y^2} = \frac{x^2}{2} \qquad \cos z \sim 1 - z^2/2$$

$$\frac{\partial f}{\partial x}(x,y) = \frac{\operatorname{sen} xy}{y}, \qquad \frac{\partial f}{\partial y}(x,y) = \frac{x \sin xy}{y^2} - \frac{2(1-\cos xy)}{y^3} \qquad y \neq 0$$

$$\frac{\partial f}{\partial x}(x,0) = \lim_{h\to 0} \frac{f(x+h,0)-f(x,0)}{h} = \lim_{h\to 0} \frac{\frac{(x+h)^2}{2} - \frac{x^2}{2}}{h} = \lim_{h\to 0} \frac{1}{2}\left(\frac{2x\cancel{h}+h^2}{\cancel{h}}\right)$$

$$= x$$

$$\frac{\partial f}{\partial y}(x,0) = \lim_{h\to 0} \frac{f(x,h)-f(x,0)}{h} = \lim_{h\to 0} \frac{\frac{1-\cos(xh)}{h^2} - \frac{x^2}{2}}{h} = 0$$

$$= \lim_{h\to 0} \frac{\frac{x^2}{2} - \frac{x^2}{2} + o(\ )}{h}$$

$$\cos z = 1 - \frac{z^2}{2} + \frac{z^4}{4!} + \dots$$

$$= \lim_{h\to 0} \frac{\cancel{\frac{x^2}{2}} - \frac{x^4 h^{\cancel{4}\,2}}{4!\,\cancel{h^2}} - \cancel{\frac{x^2}{2}}}{\cancel{h}} = \lim_{h\to 0} - \frac{x^4 h}{4!} = 0$$

$$\lim_{(x,y)\to(a,0)} \frac{\partial f}{\partial x}(x,y) = \frac{\partial f}{\partial x}(a,0) = a$$

$$\lim_{(x,y)\to(a,0)} \frac{\partial f}{\partial x} = \lim_{(x,y)\to(a,0)} \frac{\operatorname{sen} xy}{y} = \lim_{(x,y)\to(a,0)} \frac{x\cancel{y}}{\cancel{y}} = a$$

$$\operatorname{sen} z = z - \frac{z^3}{6} + \dots$$

$$\lim_{(x,y)\to(a,0)} \frac{\partial f}{\partial y} = \lim_{(x,y)\to(a,0)} \frac{x \sin xy}{y^2} - \frac{2(1-\cos xy)}{y^3}$$

$$1-\cos z = \frac{z^2}{2} - \frac{z^4}{4!} + \dots$$

$$\frac{x\, xy}{y^2} \cancel{- \frac{2(xy)^2}{2y^3}} = \frac{x^2}{y} \cancel{- \frac{x^2}{y}} = 0$$ 1° orden no vale!!

2° orden $$\lim_{(x,y)\to(a,0)} \left[\frac{x\left(\cancel{xy} - (xy)^3/6\right)}{y^2} - \frac{2}{y^3}\left(\cancel{\frac{(xy)^2}{2}} - \frac{(xy)^4}{4!} \right) \right]$$

$$= \lim_{(x,y)\to(a,0)} x\,\frac{x^3}{6}\,y + \frac{2}{4!}\,x^4 y = 0$$

$f \in C^1(\mathbb{R}^2)$. $\Rightarrow$ f es diferenciable!!

$\boxed{1.30 + 1.34.}$

$$f(x,y) = \begin{cases} \dfrac{x^3y^3}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases} \qquad \boxed{f \in C^2(\mathbb{R}^2)} \ ?$$

$$\lim_{(x,y)\to(0,0)} f(x,y) = 0 \qquad \left| \frac{x^3y^3}{x^2+y^2} \right| \leq \underbrace{\frac{|xy|}{x^2+y^2}}_{\leq 1/2} \cdot \underbrace{|xy|^2}_{\downarrow\, 0}$$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{x\to 0} \frac{\overset{0}{\cancel{f(x,0)}} - \overset{0}{\cancel{f(0,0)}}}{x} = 0 = \frac{\partial f}{\partial y}(0,0)$$

$$\frac{\partial f}{\partial x}(x,y) = \frac{x^2y^3(3y^2+x^2)}{(y^2+x^2)^2} \qquad \frac{\partial f}{\partial y}(x,y) = \frac{x^3y^2(3x^2+y^2)}{(x^2+y^2)^2}$$

$(x,y) \neq (0,0)$

$$0 \stackrel{?}{=} \lim_{(x,y)\to 0} \frac{\partial f}{\partial x}(x,y) = 0 \qquad \left| \frac{x^2y^3(3y^2+x^2)}{(y^2+x^2)^2} \right| = \left(\frac{xy}{x^2+y^2}\right)^2 |y|\,|3y^2+x^2|$$

$$\le 1/2 \qquad \to 0$$

$$\lim_{(x,y)\to 0} \frac{\partial f}{\partial y}(x,y) = 0$$

$$\Rightarrow f \in C^1(\mathbb{R}^2)$$

$$\frac{\partial^2 f}{\partial x^2}(0,0) = \lim_{x\to 0} \frac{\cancel{\frac{\partial f}{\partial x}(x,0)} - \cancel{\frac{\partial f}{\partial x}(0,0)}}{x} = 0 \qquad \frac{\partial^2 f(0,0)}{\partial y \partial x} = \lim_{(x,y)\to 0} \frac{\frac{\partial f}{\partial x}(0,y) - \cancel{\frac{\partial f}{\partial x}(0,0)}}{y} = 0$$

$$\frac{\partial^2 f}{\partial y^2}(0,0) = \lim_{y\to 0} \frac{\cancel{\frac{\partial f}{\partial y}(0,y)} - \cancel{\frac{\partial f}{\partial y}(0,0)}}{y} = 0 \qquad \frac{\partial^2 f(0,0)}{\partial x \partial y} = \lim_{(x,y)\to 0} \frac{\cancel{\frac{\partial f}{\partial y}(x,0)} - \cancel{\frac{\partial f}{\partial y}(0,0)}}{x}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{2xy^5(3y^2-x^2)}{(y^2+x^2)^3} \qquad \frac{\partial^2 f}{\partial y^2} = 2y\frac{x^5(3x^2-y^2)}{(x^2+y^2)^3}$$

$$\frac{\partial f(x,y)}{\partial x \partial y} = \frac{x^2y^2\,(3y^4 + 14x^2y^2 + 3x^4)}{(x^2+y^2)^3} \to 0$$

$$\frac{\partial^2 f}{\partial y^2} \xrightarrow{(x,y)\to 0} 0$$

$$= \left| \frac{2xy^5\,(3y^2 - x^2)}{(x^2+y^2)^3} \right| \leq 2\, \underbrace{\frac{|xy|}{(x^2+y^2)}}_{\leq 1/2}\, \underbrace{\frac{y^4}{(\cancel{x^2}+y^2)^2}}_{\leq 1}\, \underset{\downarrow\atop 0}{(3y^2+x^2)} \to 0$$

$\leq 1/4$

$$\left| \frac{\partial f(x,y)}{\partial x \partial y} \right| = \frac{x^2y^2}{(x^2+y^2)^2} \cdot \frac{3y^4 + 4x^2y^2 + 3x^4}{x^2+y^2} \leq \frac{1}{4}\left[\frac{3y^4+4x^2y^2}{\cancel{x^2}+y^2} + \frac{3x^4}{x^2+\cancel{y^2}} \right]$$

$$\frac{1}{4}\left[(3y^2+4x^2) + (3x^2) \right] \to 0$$

$\downarrow$ 0 $(x,y)\to 0$ $\Rightarrow$ $f \in C^2(\mathbb{R}^2)$

1.36

$$f(x,y) := \frac{xy(x^2-y^2)}{x^2+y^2} \quad (x,y)\neq(0,0)$$

$$f(0,0)=0$$

$$\frac{d}{dx} f(x,y) = -\frac{y(y^4-4x^2y^2-x^4)}{(y^2+x^2)^2}$$

$$|f(x,y)| = |xy| \left|\frac{x^2-y^2}{x^2+y^2}\right| \leq |xy| \to 0$$

$$\frac{d}{dy} f(x,y) = -\frac{x(y^4+4x^2y^2-x^4)}{(y^2+x^2)^2}$$

$$\underset{1}{\wedge}$$

$$f \in C^1(\mathbb{R}^2)$$

$$\frac{\partial f}{\partial y}(0,0) = 0 = \frac{\partial f}{\partial x}(0,0)$$

$$+x^4+y^4 \qquad 2(x^4+2x^2y^2+y^4)$$

$$\Rightarrow$$

$$\lim_{(x,y)\to 0} \frac{\partial f}{\partial x}(x,y) = 0$$

$$\left|\frac{\partial f}{\partial x}\right| \leq |y| \frac{x^4+4x^2y^2+y^4}{(x^2+y^2)^2} \leq |y| \frac{2\cancel{(x^2+y^2)^2}}{\cancel{(x^2+y^2)^2}} \to 0$$

$$|y|\left[\frac{(x^2+y^2)^2+2x^2y^2}{(x^2+y^2)^2}\right] \leq |y|\Big[1+2\Big(\underbrace{\frac{|xy|}{x^2+y^2}}_{\leq 1/2}\Big)^2\Big] \leq 2|y| \to 0$$

$$\frac{d^2}{dx^2}f(x,y)=\frac{4xy^3(3y^2-x^2)}{(y^2+x^2)^3}$$

$y = mx$

$$\frac{4 x m^3 \cancel{x^3} (3m^2 \cancel{x^2} - \cancel{x^2})}{(m^2+1)^3 \cancel{x^6}} \to \text{N.L.C.}$$

$$\frac{d^2}{dy^2}f(x,y)=\frac{4x^3y(y^2-3x^2)}{(y^2+x^2)^3}$$

$$\frac{d^2}{dx\,dy}f(x,y) = -\frac{(y-x)(y+x)(y^4+10x^2y^2+x^4)}{(y^2+x^2)^3}$$

$f \in C^2(\mathbb{R}^2 \setminus \{(0,0)\})$

$$\frac{\partial^2 f}{\partial x \partial y}(0,0) = \lim_{x \to 0} \frac{\frac{\partial f}{\partial y}(x,0) - \cancel{\frac{\partial f}{\partial y}(0,0)}^{\,0}}{x} = \lim_{x \to 0} \frac{x}{x} = 1$$

$\#$

$$\frac{\partial^2 f}{\partial y \partial x}(0,0) = \lim_{y \to 0} \frac{\frac{\partial f}{\partial x}(0,y)^{\,x} - \cancel{\frac{\partial f}{\partial x}(0,0)}^{\,0}}{y} = \lim_{y \to 0} \frac{-y}{y} = -1$$

$$\frac{x\,y^2}{y^2+2\,x\,y+x^2}$$

$$(x,y)\to 0$$

$$\frac{xy^2}{(x+y)^2}$$

$$x=-y+y^2$$

$$\frac{(-y+y^2)\,y^2}{y^4}=\frac{-y^3+y^4}{y^4}=-\frac{1}{y}+1$$

No acotado