$$f(x,y) := x\,y\,\sin\left(\frac{\frac{1}{2}(x-y)}{x+y}\right)$$

$x+y=0$ $\quad (a,-a)$ $\quad a\in\mathbb{R}$ $\quad \lim_{(x,y)\to 0} f = 0$

$f(a,-a)=0$

$a\neq 0$ $\quad f(x,y) \sim -a^2 \sin\frac{a}{0}$ $\not\to$ N.T.L.

$$\frac{\partial f}{\partial x}(0,0) = \lim_{x\to 0} \frac{\cancel{f(x,0)}^{\,0} - \cancel{f(0,0)}^{\,0}}{x} = 0 = \lim_{y\to 0}\frac{\cancel{f(0,y)}^{\,0} - \cancel{f(0,0)}^{\,0}}{y} = \frac{\partial f}{\partial y}(0,0)$$

$$\left|\frac{f(x,y) - \cancel{f(0,0)}^{\,0} - \cancel{Df(0,0)\begin{pmatrix}x\\y\end{pmatrix}}^{\,0}}{\sqrt{x^2+y^2}}\right| \quad \frac{|xy|}{\sqrt{x^2+y^2}}\left|\underbrace{\sin\left(\frac{1}{2}\,\frac{x-y}{x+y}\right)}_{}\right|$$

≤ 1

$\frac{|xy|}{\sqrt{x^2+y^2}} \downarrow 0$ $\quad$ acot. $\sqrt{\frac{|xy|}{x^2+y^2}}$ $\quad \sqrt{xy}\downarrow 0$

oscil.

$$\frac{x y^2}{(y+x)^2}$$

$y_n = \frac{1}{n}$ $\quad x_n = -\frac{1}{n} + \frac{1}{n^2}$ $\quad (x_n, y_n) \to 0$

$$\frac{\left(-\frac{1}{n}+\frac{1}{n^2}\right)\frac{1}{n^2}}{\left(\frac{1}{n^2}\right)^2} = \frac{n^{\cancel{4}\,2}}{\cancel{n^2}}\left(-\frac{1}{n}+\frac{1}{n^2}\right) = \underline{\underline{-n+1}}$$

No acotado

$\frac{\partial f}{\partial x}$ no es cont.