T. H-4 $\quad \Delta f(a,b) = [f(a+h, b+h) - f(a+h, b)] - [f(a, b+h) - f(a,b)]$

$G(x) = f(x, b+h) - f(x,b)$ $\quad h \to t_q \quad U = \{(x,y) \mid x \in (a, a+h),\ y \in (b, b+h)\} \subseteq A$

$\Delta f(a,b) = G(a+h) - G(a)$ $\quad x \in [a, a+h]$ aplic TVMC.

$$\Delta f(a,b) = G'(\xi)\, h = \left[\frac{\partial f}{\partial x}(\xi, b+h) - \frac{\partial f}{\partial x}(\xi, b)\right] h$$

$$\left[\tilde f(x,y) = \tilde f(a,b) + \frac{\partial \tilde f}{\partial x}(a,b)(x-a) + \frac{\partial \tilde f}{\partial y}(a,b)(y-b) + o\left(\sqrt{(x-a)^2 + (y-b)^2}\right)\right]$$

$$\frac{\partial f}{\partial x}(\xi, b+h) = \frac{\partial f}{\partial x}(a,b) + \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}(a,b)\right)(\xi - a) + \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}(a,b)\right) h + o\big(\underbrace{\sqrt{(\xi-a)^2 + h^2}}_{o(h)}\big)$$

$$\frac{\partial f}{\partial x}(\xi, b) = \frac{\partial f}{\partial x}(a,b) + \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}(a,b)\right)(\xi - a) + \cancel{(\quad)}\, 0 + o\big(\underbrace{\sqrt{(\xi-a)^2}}_{o(h)}\big)$$

$\xi \in (a; a+h) \qquad \xi - a = \alpha h \qquad \alpha \in (0,1) \qquad o(\alpha h) = o(h)$

$\sqrt{\alpha^2 h^2 + h^2} = \sqrt{\alpha^2 + 1}\, |h|$

$$\Delta f(a,b) = \frac{\partial^2 f(a,b)}{\partial y \partial x} \cdot h^2 + h\,o(h) \Rightarrow \lim_{h\to 0} \frac{\Delta f(a,b)}{h^2} = \frac{\partial^2 f(a,b)}{\partial y \partial x} + \cancelto{0}{\frac{o(h)}{h}}$$

$$H(y) = f(a+h,y) - f(a,y) \qquad \Delta f(a,b) = H(b+h) - H(b) \Rightarrow \text{T.V.M L.}$$

$$\Delta f(a,b) = H'(\xi)\,h = \left[\frac{\partial f}{\partial y}(a+h,\xi) - \frac{\partial f}{\partial y}(a,\xi)\right] h \qquad \xi \in (b, b+h)$$

$$\frac{\partial f}{\partial y}(a+h,\xi) = \frac{\partial f}{\partial y}(a,b) + \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}(a,b)\right) h + \frac{\partial}{\partial y}\left[\frac{\partial f}{\partial y}(a,b)\right](\xi - b) + o\left(\sqrt{h^2 + (\xi-b)^2}\right) \overset{\shortparallel}{\ } o(h)$$

$$\frac{\partial f}{\partial y}(a,\xi) = \frac{\partial f}{\partial y}(a,b) + (\quad)\,0 + \frac{\partial}{\partial y}\left[\frac{\partial f}{\partial y}(a,b)\right](\xi - b) + \underset{o(\xi-b)}{\underset{\shortparallel}{o(h)}}$$

$$\lim_{h\to 0} \frac{\Delta f(a,b)}{h^2} = \frac{\partial^2 f(a,b)}{\partial x \partial y} + \cancelto{0}{\frac{o(h)}{h}} \qquad \Rightarrow \frac{\partial^2 f(a,b)}{\partial x \partial y} = \frac{\partial^2 f(a,b)}{\partial y \partial x}$$

$- \ f(x,y) = x^2 y \,\text{sen}\,\frac{1}{x} \qquad x \neq 0 \qquad f(x,y) = 0 \quad \text{si} \quad x = 0$

$\frac{\partial f}{\partial x}(0,y) = 0, \quad \frac{\partial f}{\partial x} = 2xy\,\text{sen}\,\frac{1}{x} - y\cos\frac{1}{x}, \quad \frac{\partial f}{\partial y}(0,y) = 0, \quad \frac{\partial f}{\partial y} = x^2\,\text{sen}\,\frac{1}{x} \qquad x \neq 0$

$$\frac{\partial^2 f}{\partial x \partial y} = 2x\,\text{sen}\,\frac{1}{x} - \cos\frac{1}{x} \qquad \underline{(0,0)}$$

$$\frac{\partial^2 f}{\partial x \partial y}(0,0) = \frac{\partial}{\partial x}\left[\frac{\partial f}{\partial y}(0,0)\right] = \lim_{x \to 0} \frac{\frac{\partial f}{\partial y}(x,0) - \cancel{\frac{\partial f}{\partial y}(0,0)}^{\,0}}{x} = \lim_{x \to 0} \frac{x^{\cancel{2}}\,\text{sen}\,1/x}{\cancel{x}} = 0$$

$$\frac{\partial^2 f}{\partial y \partial x}(0,0) = \frac{\partial}{\partial y}\left[\frac{\partial f}{\partial x}(0,0)\right] = \lim_{y \to 0} \frac{\frac{\partial f}{\partial x}(\cancel{0,y}) - \frac{\partial f}{\partial x}\cancel{(0,0)}^{\,0}}{y} = 0$$

$$\frac{\partial^2 f}{\partial x \partial y}(0,0) = \frac{\partial^2 f}{\partial y \partial x}(0,0)$$