$\mathcal{D}: \mathbb{P} \to \mathbb{P}$ $\quad : \frac{d}{dx} := D.$ $\quad D p_n = \frac{d}{dx} p_n$

$\forall \alpha, \beta \in \mathbb{R} \quad D(\alpha p_n + \beta q_n) = \alpha D p_n + \beta D q_n \in \mathbb{P}$

$T: \mathbb{R}^n \to \mathbb{R}^m \quad y = Ax \quad x \in \mathbb{R}^n \quad y \in \mathbb{R}^m \quad A = \mathbb{R}^{m \times n}$

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$A(x+y) = Ax + Ay \qquad A(\alpha x) = \alpha(Ax)$

- $\|T\| = \sup\limits_{x \neq 0} \frac{\|Tx\|}{\|x\|} \Rightarrow \|I\| = \sup\limits_{x \neq 0} \frac{\|Ix\|}{\|x\|} = \sup\limits_{x \neq 0} \frac{\|x\|}{\|x\|} = 1$

$\|\Theta\| = \sup\limits_{x \neq 0} \frac{\|\Theta x\|}{\|x\|} = \sup\limits_{x \neq 0} \frac{\|0\|}{\|x\|} = 0$

Teorema: $\dim(X) = n \quad \forall T$ lineal es tal que $\|T\| < +\infty$

$\forall x \in X \quad \exists c \geq 0, \|Tx\| \leq c\|x\|$. Como X es dim finita $\exists e_1 \cdots e_n$

$x = \alpha_1 e_1 + \cdots + \alpha_n e_n \qquad \|Tx\| = \|T \sum\limits_k \alpha_k e_k\| = \|\sum\limits_k \alpha_k Te_k\|$

$\leq \sum\limits_k |\alpha_k| \|Te_k\| \leq M \cdot \sum\limits_k |\alpha_k| \qquad M = \max\limits_k \|Te_k\|$

$$\exists \gamma > 0$$

$$\|x\| = \|\sum \alpha_k e_k\| \geq \gamma \left(\sum_k |\alpha_k|\right) \qquad \sum |\alpha_k| \leq \frac{\|x\|}{\gamma}$$

$$\|Tx\| \leq \underbrace{\frac{M}{\gamma}}_{c} \|x\| = c\,\|x\| \qquad \exists c \geq 0 \text{ t.q. } \|Tx\| \leq c\,\|x\| \quad \forall x \in X.$$

$*$ $(x_n)_n \in X$ $\quad x_n \xrightarrow{n} x \in X$ $\quad \lim_{n\to\infty} T x_n = T \lim_n x_n = Tx$

$\forall \varepsilon > 0 \quad \exists \delta > 0$ t.q. $\|x - x_0\| < \delta \Rightarrow \|Tx - Tx_0\| < \varepsilon.$

T lineal acotado. $\Rightarrow$ T es cont en $A \subset X$. $\qquad T : A \subset X \to Y$

$\forall x_0 \in A \qquad \|Tx - Tx_0\| = \|T(x - x_0)\| \leq \|T\|\,\|x - x_0\| \leq \|T\|\,\delta = \varepsilon$

$\forall \varepsilon > 0 \quad \exists \delta < \varepsilon/\|T\| \qquad \|T\| \neq 0 \qquad \|T\| = 0 \Rightarrow T = 0$

$\Leftarrow$ cont. en x_0 $\quad \forall \varepsilon > 0 \quad \exists \delta > 0 \quad \|x - x_0\| < \delta \Rightarrow \|Tx - Tx_0\| < \varepsilon$

$\forall y \neq 0 \quad x = x_0 + \frac{\delta}{2\|y\|} y \qquad \|x - x_0\| = \left\|\frac{\delta y}{2\|y\|}\right\| = \frac{\delta}{2} < \delta^{>0}$

T cont en x_0 $\Rightarrow$ $\forall \varepsilon > 0 \; \exists \delta$ $\|Tx - Tx_0\| < \varepsilon$ $\quad \forall y \neq 0$

$$\varepsilon > \|T(x - x_0)\| = \left\| T\left(\frac{\delta}{2\|y\|} y\right) \right\| = \frac{\delta}{2\|y\|} \|Ty\| < \varepsilon \Rightarrow$$

$$\|Ty\| < \frac{2\varepsilon}{\delta} \|y\| \quad \forall y \neq 0 \quad \Rightarrow T \text{ es acotado}$$

$$f(x+h) - f(x) - Df(x)(h) = o(\|h\|) \qquad o(\|h\|) = \lim_{h \to 0} \frac{o(\|h\|)}{\|h\|} = 0$$

$$\underbrace{\begin{pmatrix} f_1(x+h) \\ \vdots \\ f_m(x+h) \end{pmatrix}}_{\mathbb{R}^m} - \begin{pmatrix} f_1(x) \\ \vdots \\ f_m(x) \end{pmatrix} - \underbrace{\begin{pmatrix} & A & \end{pmatrix}}_{\mathbb{R}^{m \times n}} \underbrace{\begin{pmatrix} h_1 \\ \vdots \\ h_n \end{pmatrix}}_{\mathbb{R}^n} = \frac{o(\|h\|)}{\|h\|} = \begin{pmatrix} o_1(\|h\|) \\ \vdots \\ o_m(\|h\|) \end{pmatrix}$$

$$f_k(x+h) - f_k(x) - (k\text{-fila de } A) \begin{pmatrix} h_1 \\ \vdots \\ h_n \end{pmatrix} = o(\|h\|) \; k = 1, \dots, m \quad = \frac{o_k(\|h\|)}{\|h\|} \to 0$$

$$\| f(x+h) - f(x) \| = \| Df(x)(h) + o(\|h\|) \| \leq$$

$$\leq \| Df(x) \| \, \|h\| + o(\|h\|) \to 0$$

$$\lim_{h \to 0} \| f(x+h) - f(x) \| = 0 \iff \lim_{h \to 0} f(x+h) = f(x)$$

- f diferenciable en x_0

$$D_u f(x_0) = \lim_{\lambda \to 0} \frac{f(x_0 + \lambda u) - f(x_0)}{\lambda} = \lim_{\lambda \to 0} \frac{Df(x_0)(\lambda u) + o(\|\lambda u\|)}{\lambda}$$

$$= \lim_{\lambda \to 0} \frac{\cancel{\lambda}\, Df(x_0)(u)}{\cancel{\lambda}} + \cancel{\frac{o(|\lambda|)}{\lambda}} \qquad \left| \frac{o(|\lambda|)}{|\lambda|} \right| \xrightarrow{\lambda \to 0} 0$$

$$D_{e_i} f(x_0) = Df(x_0)(e_i) = A \begin{pmatrix} 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \end{pmatrix} \text{ i-posición} = \text{i-ésima columna de } A$$

$$\frac{\partial f}{\partial x_i}(x_0) \qquad i = 1, 2, \dots n$$

$$f(x,y) = \begin{cases} \frac{x^3 y}{x^4+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = 0 \end{cases}$$

cont. en $(0,0)$

$\|u\|^2 = u_x^2 + u_y^2 = 1$

$$\frac{f(0+\lambda u_x, 0+\lambda u_y) - \cancelto{0}{f(0,0)}}{\lambda} = \frac{\lambda^3 u_x^3 \cdot \lambda u_y}{\lambda^4 u_x^4 + \lambda^2 u_y^2} = \frac{\lambda^{\cancel{4}\,2}}{\cancel{\lambda^2}} \cdot \frac{u_x^3 u_y}{\lambda^2 u_x^4 + u_y^2}$$

$\lambda \to 0 \quad 0 \cdot \frac{u_x^3 \cancel{u_y}}{u_y^{\cancel{2}}}$ $\quad u_y \neq 0 \quad \Big| \quad u_y = 0$

$D_u f(0,0) = 0$

$$\frac{\partial f(0,0)}{\partial x} = \lim_{x \to 0} \frac{\overset{=0}{f(x,0)} - \cancelto{0}{f(0,0)}}{x} = 0 \qquad \frac{\partial f}{\partial y}(0,0) = \lim_{y \to 0} \frac{\overset{=0}{f(0,y)} - \cancelto{0}{f(0,0)}}{y} = 0$$

$$Df(0,0) \overset{?}{=} (0 \quad 0) = \left(\frac{\partial f(0,0)}{\partial x} , \frac{\partial f(0,0)}{\partial y} \right)$$

$$\frac{f(x,y) - \cancel{f(0,0)}^{0} - \cancel{Df}(0,0)^{0}\begin{pmatrix} x \\ y \end{pmatrix}}{\sqrt{x^2+y^2}} \xrightarrow[(x,y)\to 0]{?} 0$$

$$\frac{x^3y}{(x^4+y^2)\sqrt{x^2+y^2}} = \frac{x}{\sqrt{x^2+y^2}} \frac{x^2y}{x^4+y^2} \qquad y = mx$$

$$\frac{x}{\sqrt{x^2+m^2x}} \frac{x^2mx}{x^4+m^2x^2} = \frac{m}{\sqrt{1+m^2}} \frac{x^{\cancel{4}\,2}}{(\cancel{x}^2+m^2)\,|x|\,\cancel{x^2}} \to 0$$

$$y = x^2 \qquad \frac{x^5}{(x^4+x^4)\sqrt{x^2+x^2}} = \frac{x}{2\sqrt{2x^2}} = \frac{1}{4}\frac{x}{|x|} \not\to 0$$