$$D_u f(a,b) = Df(a)(u) = \left(\frac{\partial f}{\partial x_1}(a) \ \dots \ \frac{\partial f}{\partial x_n}(a)\right)\begin{pmatrix}u_1\\ u_2\\ \vdots\\ u_n\end{pmatrix} =$$

$f:\mathbb{R}^n \to \mathbb{R}$

$$= \frac{\partial f}{\partial x_1}(a)\,v_1 + \dots + \frac{\partial f}{\partial x_n}(a)\,v_n = \langle \nabla f(a), u\rangle$$

$f:\mathbb{R}^2 \to \mathbb{R}$

$$f(x,y) - f(a,b) - Df(a,b)\begin{pmatrix}x-a\\ y-b\end{pmatrix} = o(\|h\|)$$

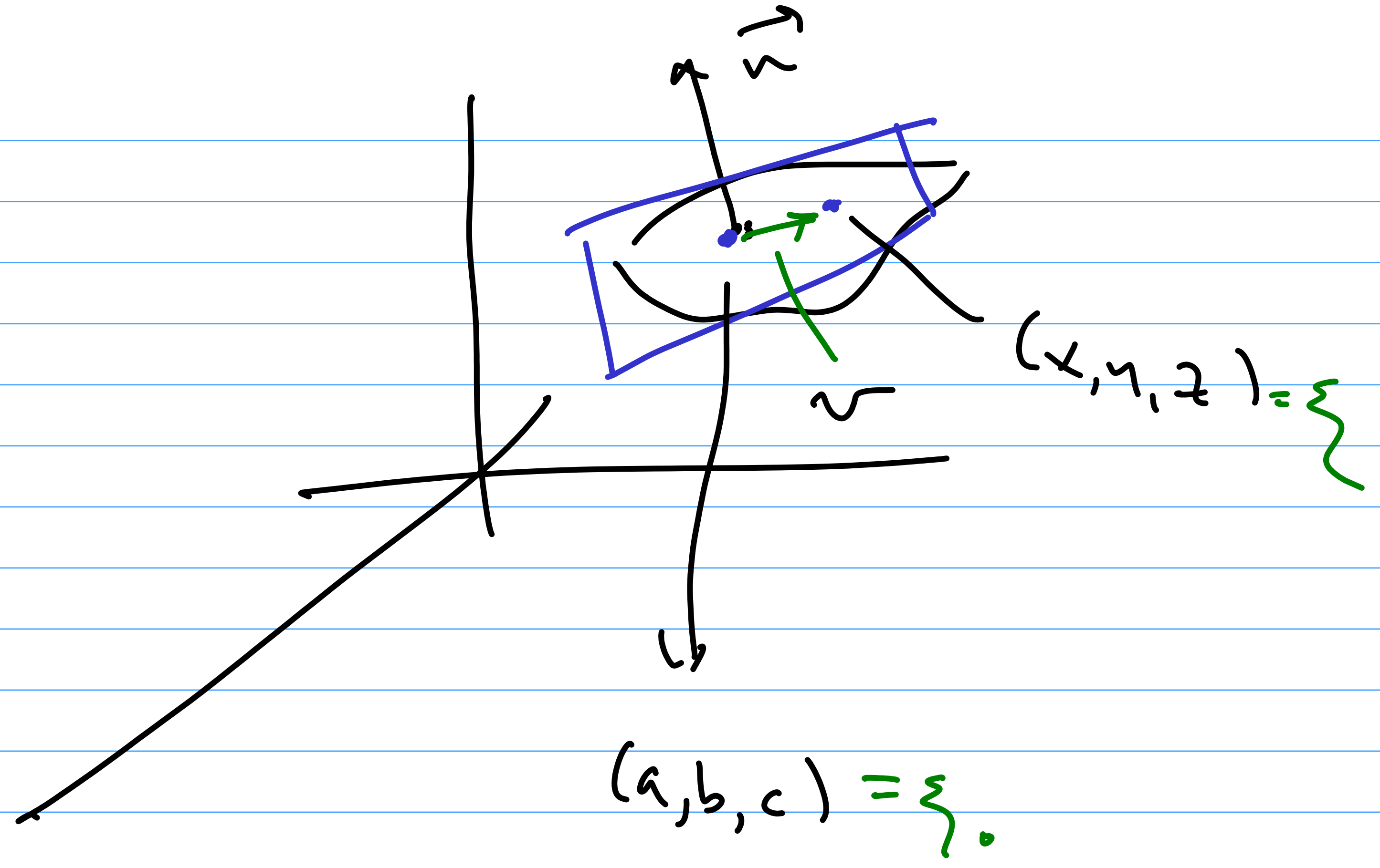
$$f(x,y) - f(a,b) - \frac{\partial f}{\partial x}(a,b)(x-a) - \frac{\partial f}{\partial y}(a,b)(y-b) = o(\|h\|)$$

$$\|h\| = \sqrt{(x-a)^2 + (x-b)^2}$$

$$\vec{n} = \left(\frac{\partial f}{\partial x}(a,b), \frac{\partial f}{\partial y}(a,b), -1\right)$$

$$\langle \vec{n}, (x-a, y-b, -1)\rangle = 0$$

$$\cos\theta = \frac{\langle \vec{v}, \vec{n} \rangle}{\|\vec{v}\| \, \|\vec{n}\|} \to 0 \qquad \xi \to \xi_0$$



$$\langle \vec{v}, \vec{n} \rangle = \frac{\partial f}{\partial x}(\xi_0)(x-a) + \frac{\partial f}{\partial y}(\xi_0)(y-b) - (z-c)$$

$$= \frac{\partial f}{\partial x}(a,b)(x-a) + \frac{\partial f}{\partial y}(a,b)(y-b) - f(x,y) + f(a,b)$$

$$\vec{v} = (x-a, y-b, z-c)$$

$$= -\left(f(x,y) - f(a,b) - \frac{\partial f}{\partial x}(a,b)(x-a) - \frac{\partial f}{\partial y}(a,b)(y-b) \right)$$

$$\vec{n} = \left(\frac{\partial f}{\partial x}(\xi_0), \frac{\partial f}{\partial y}(\xi_0), -1 \right)$$

$$= - o(\|h\|)$$

$$z = f(x,y)$$

$$|\cos\theta| = \frac{|o(\|h\|)|}{\|\vec{v}\|\,\|\vec{n}\|} \leq \frac{|o(\|h\|)|}{1\,\|h\|} \to 0 \qquad \|\vec{n}\| \geq 1 \qquad \|\vec{v}\| = \sqrt{(x-a)^2+(x-b)^2} \geq \|h\|$$

$$\cos\theta \longrightarrow 0 \qquad (x,y) \to (a,b) \qquad \theta = \pi/2$$

$$f(x,y) = \sqrt{1-x^2-y^2}$$

$$\xi_0 = \left(\sqrt{2}/2, 1/2, 1/2\right)$$

$$\vec{v} = \left(x - \sqrt{2}/2, y - 1/2\right)$$

$$\langle \vec{n}, \vec{v} \rangle = 0$$

$$\vec{n} = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, -1\right)\Bigg|_{(\sqrt{2}/2,\,1/2)}$$

$$\frac{\partial f}{\partial y} = \frac{-2y}{2\sqrt{1-x^2-y^2}}\Bigg|_{(\sqrt{2}/2,\,1/2)} = -1$$

$$\frac{\partial f}{\partial x} = \frac{-2x}{2\sqrt{1-x^2-y^2}}\Bigg|_{(\sqrt{2}/2,\,1/2)} = -\sqrt{2}$$

$$(-\sqrt{2}, -1, -1)\left(x - \frac{\sqrt{2}}{2}, y - 1/2, z - 1/2\right) = 0$$

$$+\sqrt{2}\left(x - \frac{\sqrt{2}}{2}\right) + (y - 1/2) + 1(z - 1/2) = 0$$

$f, g : \mathbb{R}^n \to \mathbb{R}$

$D(f \cdot g)$

$= o(\|h\|)$

$$f(a+h)g(a+h) - f(a)g(a) = f(a+h)g(a+h) - f(a+h)g(a) + f(a+h)g(a) - f(a)g(a)$$

$$= f(a+h)\left[g(a+h) - g(a)\right] + \left[f(a+h) - f(a)\right]g(a) =$$

$$= f(a+h)\left[Dg(a)(h) + o(\|h\|)\right] + \left[Df(a)(h) + o(\|h\|)\right]g(a)$$

$$= \left[f(a) + Df(a)(h) + o(\|h\|)\right]\left[Dg(a)(h) + o(\|h\|)\right] +$$

$$+ \quad Df(a)(h) \cdot g(a) + g(a)\, o(\|h\|) =$$

$$= f(a)\, Dg(a)(h) + Df(a)(h)\, g(a) +$$

$$\Big(Df(a)(h)\, Dg(a)(h) + Df(a)(h)\, o(\|h\|) + Dg(a)(h)\, o(\|h\|) +$$

$$+ o(\|h\|)\, o(\|h\|) \Big) \qquad (\ ) = o(\|h\|)$$

$$f(a+h)\,g(a+h) - f(a)\,g(a) - \underbrace{f(a)\,Dg(a)(h) + g(a)\,Df(a)(h)} + o(\|h\|)$$

$$D f\cdot g(a) = [f(a)\, Dg(a) + g(a)\, Df(a)](h)$$

$$\| Df(a)(h) \cdot Dg(a)(h) \| = \| Df(a)(h) \| \, \| Dg(a)(h) \| \leq$$

$$\leq \| Df(a) \| \, \|h\| \, \| Dg(a) \| \, \|h\| \leq M \, \|h\|^2 = o(\|h\|)$$

$$\| T \, o(\|h\|) \| \leq \|T\| \, o(\|h\|) = o(\|h\|)$$

$$o(\|h\|) \cdot o(\|h\|) = o(\|h\|)$$

$$\frac{o(\|h\|) \, o(\|h\|)}{\|h\|} \to 0$$