$$f(x_1 \dots x_n) = \begin{pmatrix} f_1(x_1 \dots x_n) \\ \vdots \\ f_m(x_1 \dots x_n) \end{pmatrix}$$

$$\| f - L \| < \varepsilon \Rightarrow$$

$$\sum_{k=1}^{n} | f_k(x_1 \dots x_n) - L_k |^2 < \varepsilon^2$$

$$f_k \to L_k$$

$$\|f\| = \|f - L + L\| \leq \|f - L\| + \|L\| \left\{ -\|f-L\| \leq \|f\| - \|L\| \leq \underbrace{\|f - L\|}^{<1} \right.$$

$\forall \varepsilon > 0 \ (\varepsilon = 1)$

$\exists \delta > 0 \quad 0 < \|x - x_0\| < \delta$

$$\big| \|f\| - \|L\| \big| < 1$$

$$\underset{1}{\underset{\shortparallel}{L - \varepsilon}} \leq \|f\| \leq \underset{1}{\underset{\shortparallel}{L + \varepsilon}}$$

- A compacto f continua en A. $\Rightarrow$ f está acot. y alcanza max y min absolutos.

1) Supongamos que f no es acotada. $\forall n \in \mathbb{N}$ $\exists x_n \in A$ $|f(x_n)| > n$

$\exists (x_n)_n \subset A$ tq $f(x_n)$ no acotada.

A compacto $\Rightarrow$ $x_{n_k} \to x \in A$. $f(x_{n_k}) \to f(x)$ imposible.

no acotada.

2) $S = \sup_A f(x)$ sup S no es alcanzable $\nexists x \in A$ tq $f(x) = S$.

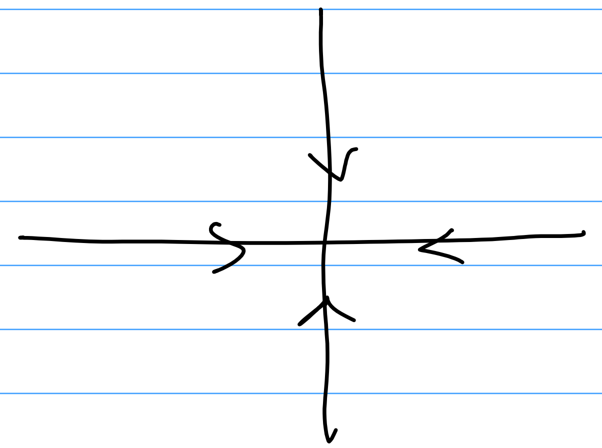
$g(x) = \dfrac{1}{S - f(x)} > 0$ $\exists M > 0$ tq $\dfrac{1}{S - f(x)} < M$

$\dfrac{1}{M} < S - f(x) \Rightarrow$ $f(x) < S - \dfrac{1}{M}$!!! S es el supremo !

$\underline{\forall x}$

1) $f(x,y)=\dfrac{x^2y}{x^4+y^2}$ $\qquad y=mx$, $m\neq 0$ $\qquad \dfrac{mx^3}{x^4+m^2x^2}$ $\qquad \dfrac{mx}{m^2+x^2}\to 0$ $\qquad \forall m\neq 0$

$y=\alpha x^2$ $\qquad f(x,y)=\dfrac{x^2\,\alpha x^2}{x^4+\alpha^2x^4}=\dfrac{\alpha}{1+\alpha^2}\not\to$ N.L.



2) $f(x,y)=\dfrac{x^3y}{x^4+y^2}$ $\qquad f(x,mx)=\dfrac{mx^4}{x^4+m^2x^2}=\dfrac{mx^2}{x^2+m^2}\to 0$

$f(x,\alpha x^2)=\dfrac{\alpha x^5}{x^4+\alpha x^4}\to 0$ $\qquad y=x^\alpha$ $\qquad \alpha>0$, $\qquad x>0$ $\qquad |x|^\alpha$

$$f(x,x^\alpha)=\frac{x^3x^\alpha}{x^4+x^{2\alpha}}\quad\begin{cases}0<\alpha<2 & \sim\dfrac{x^{3+\alpha}}{x^{2\alpha}}=x^{3-\alpha}\to 0\\[2ex] \alpha>2 & \sim\dfrac{x^{3+\alpha}}{x^4}=x^{\alpha-1}\to 0\end{cases}$$

$|f(x,y)|\xrightarrow{?}0$

$$0 \le \left| \frac{x^3 y}{x^4+y^2} \right| \le \begin{cases} 1^\circ & \left| \frac{x^3 y}{y^2} \right| = \left| \frac{x^3}{y} \right| \quad \text{N.T.L.} \qquad \text{MALA COTA.} \\ 2^\circ & \left| \frac{x^3 y}{x^4} \right| = \left| \frac{y}{x} \right| \qquad \text{MALA COTA} \\ 3^\circ & |x| \frac{|x^2||y|}{x^4+y^2} \le |x| \frac{1}{2} \xrightarrow[y\to 0]{x \to 0} 0 \end{cases}$$

$$\frac{|ab|}{a^2+b^2} \le \frac{1}{2}$$

$$\sqrt{\|x^2+y^2\|} \to 0$$

$$\Rightarrow \|f-0\| \to 0$$

$$\frac{x+y}{x^2+y^2} \qquad y = mx \qquad \frac{x+mx}{x^2+m^2x^2} \to \frac{1}{x} \frac{m}{1+m^2} \qquad \text{N. Lím.}$$

$$f(x,y) = x + y \operatorname{sen} \frac{1}{x} \qquad x \to 0 \qquad \operatorname{sen} \frac{1}{x} \ \text{N.T.L.} \qquad x \neq 0$$

$$|f(x,y)| = \left| x + y \operatorname{sen}\frac{1}{x} \right| \le |x| + |y| \underbrace{\left| \operatorname{sen} \frac{1}{x} \right|}_{\text{acotado}}^{\le 1} \le |x| + |y| \xrightarrow[x,y \to 0]{} 0$$

$f(0,y)=0 \qquad f(0,0)=0 \qquad \lim_{(x,y)\to(0;0)} f(x,y)=0$

$$\lim_{\substack{x\to 0\\ y\to a\neq 0}} f(x,y) = \lim_{\substack{x\to 0\\ y\to a}} \cancel{x}^{\,0} + \lim_{\substack{x\to 0\\ y\to a}} \cancel{y}\ \cancel{\operatorname{sen}\frac{1}{x}} \qquad \text{N.T.L.}$$

$a\neq 0$