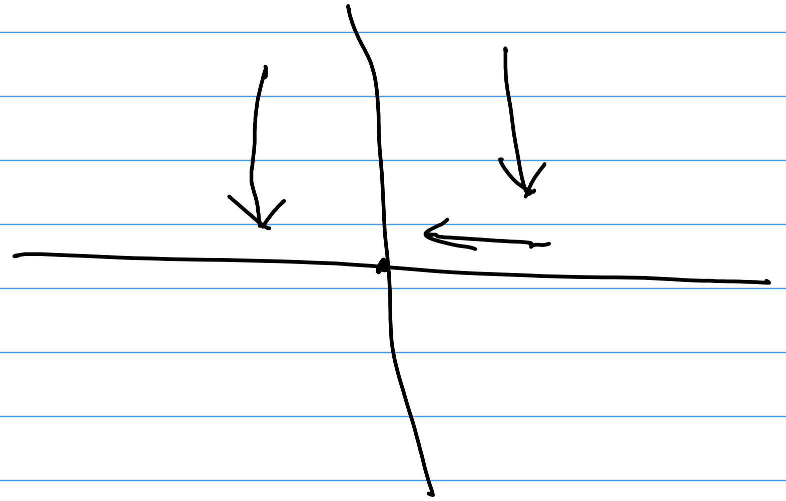
$$\lim_{(x,y)\to(0,0)} f(x,y) \qquad f(x,y) = x \operatorname{sen}\frac{1}{y} + y \operatorname{sen}\frac{1}{x} \qquad \begin{matrix} x\neq 0 \\ y \neq 0 \end{matrix}$$

$\lim_{x\to 0}\left(\lim_{y\to 0} f(x,y)\right)$ No existe. $\lim_{y\to 0}\left(\lim_{x\to 0} f(x,y)\right)$ No existe



$$\|f(x,y)\| = |f(x,y)| = \left| x \operatorname{sen}\frac{1}{y} + y \operatorname{sen}\frac{1}{x} \right| \leq$$

$$\leq |x| \left|\operatorname{sen}\frac{1}{y}\right| + |y| \left|\operatorname{sen}\frac{1}{x}\right|$$

$$\leq |x| + |y| \xrightarrow[(x,y)\to 0]{} 0$$

- $f(x,y) = \dfrac{xy}{x^2+y^2}$ $\qquad y = mx \qquad f(x,mx) = \dfrac{m}{1+m^2}$ $\qquad$ N.T.L.

$$\lim_{x\to 0}\left(\lim_{y\to 0} f(x,y)\right) = \lim_{x\to 0}\left(\lim_{y\to 0} \cancel{\frac{xy}{x^2+y^2}}^{\,0}\right) = 0 = \lim_{y\to 0}\left(\lim_{x\to 0} f(x,y)\right)$$

$m \neq 0$

$f(x,y) = \frac{y}{x} \operatorname{sen}(x^2+y^2) \qquad x \neq 0 \qquad (0,y) \qquad a \in \mathbb{R}$

$$\lim_{(x,y)\to(0,a)} \frac{y}{x} \operatorname{sen}(x^2+y^2) = \begin{cases} a = 0 \\ a \neq 0 \end{cases}$$

$a=0)$ $\quad \left| \frac{y}{x} \operatorname{sen}(x^2+y^2) \right| \sim \left| \frac{y}{x}(x^2+y^2) \right| \qquad y = mx \qquad \frac{m\cancel{x}}{\cancel{x}}(x^2+m^2x^2) \to 0$

$\#$

$\operatorname{sen} z \sim z$
$z \to 0$ $\qquad \left| \frac{y}{x}(x^2+y^2) \right| = \left| yx + \frac{y^3}{x} \right| \qquad y^3 = x \qquad |y^4+1| \to 1$

- $\lim_{x,y\to 0} \begin{pmatrix} 1/x \\ 1/y \end{pmatrix} =$ N.T.L.

- $\lim_{(x,y)\to(+\infty,+\infty)} \frac{x+2y}{x^2+2xy+2y^2} \qquad \left| \frac{(x+y)+y}{(x+y)^2+y^2} \right| = \left| \frac{(x+y)}{(x+y)^2+y^2} + \frac{y}{(x+y)^2+y^2} \right| \leq$

$x \to +\infty, \; y \to +\infty \qquad x+y \to +\infty$

$$\le \frac{|x+y|}{(x+y)^2} + \frac{|y|}{y^2} = \frac{1}{\cancel{|x+y|}}^{\to 0} + \frac{1}{\cancel{|y|}}^{\to 0} = 0$$

$x \to 1/x \qquad y \to 1/y$

$$\lim_{(x,y)\to(0^+,0^+)} \frac{\frac{1}{x} + \frac{2}{y}}{\frac{1}{x^2} + \frac{2}{xy} + \frac{2}{y^2}} = \lim_{(x,y)\to 0} \frac{(xy)(2x+y)}{2x^2+2xy+y^2}$$

$$\left|\frac{(xy)(\overbrace{2x+y}^{x+(x+y)})}{(x+y)^2+x^2}\right| = \left|\frac{x\,(z-x)\,(z+x)}{z^2+x^2}\right| = \left|\frac{z^2-x^2}{x^2+z^2}\right| |x| \le 1 \cdot |x| \xrightarrow[x,y\to 0]{} 0$$

$z = x+y \to 0$

$y = z-x$

$|z^2-x^2| \le |z^2| + |-x^2|$

$$f = \frac{x+2y}{x^2+2xy+2y^2}$$

$(x+y)^2+y^2$

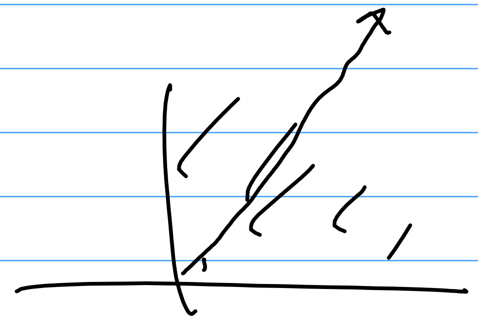
$$\lim_{x,y\to(+\infty,+\infty)} f(x,y) = \lim_{\substack{r\to\infty \\ \forall\varphi}} \frac{r\cos\varphi + 2r\sin\varphi}{(r\cos\varphi + r\sin\varphi)^2 + r^2\sin^2\varphi}$$

$x = r\cos\varphi$

$y = r\sin\varphi$

$r \to \infty$

$$= \lim_{\substack{r\to\infty \\ \forall\varphi}} \frac{1}{r} \frac{\cos\varphi + 2\sin\varphi}{(\cos\varphi + \sin\varphi)^2 + \sin^2\varphi} = \psi(\varphi)$$



$\operatorname{sen}^2\varphi = 0$ y $(\cos\varphi + \operatorname{sen}\varphi)^2 = 0$

$\downarrow$ $\downarrow$

$\varphi = k\pi$ $\cos\varphi + \operatorname{sen}\varphi = 0$

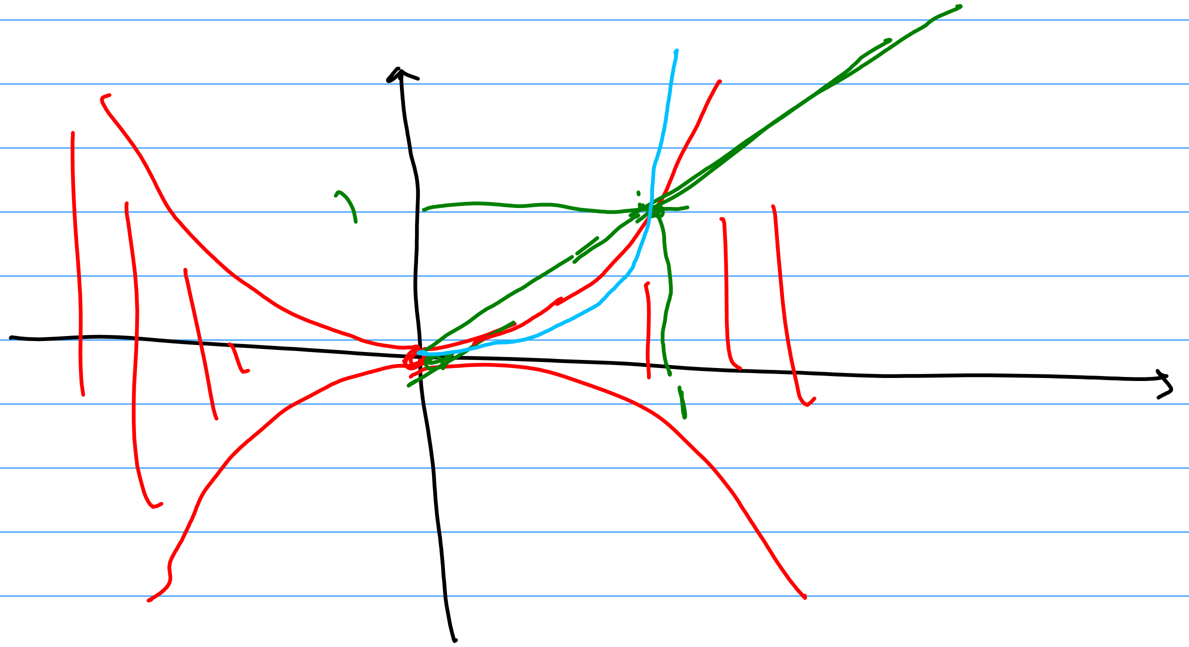
$\tan\varphi = -1$

$\Big\{ \rightarrow \psi^{-1}(\varphi)$ es acotada

$\Rightarrow \left| \frac{1}{r}\psi(\varphi) \right| \xrightarrow[r\to\infty]{} 0$

$f(x,y) = \frac{x^2-y^2}{x^2+y^2}$ $\lim_{(x,y)\to(0,0)} f(x,y) =$ N. existe ! $y = mx$

$f: D \subset \mathbb{R}^2 \to \mathbb{R}$ $D = \{(x,y) \text{ t.q. } |y| < x^2$



$x > x^2$ $x \in (0,1)$

$y = x^3$

$f(x,x^3) = \frac{x^2 - x^6}{x^2+x^6} = \frac{1-x^4}{1+x^4} \xrightarrow{x\to 0} 1$

$$|y| < x^2 \Rightarrow y^2 < x^4$$

$$\left|\frac{x^2-y^2}{x^2+y^2} - 1\right| = \left|\frac{\cancel{x^2}-y^2-\cancel{x^2}-y^2}{x^2+y^2}\right| = \frac{2y^2}{x^2+y^2} \le \frac{2x^4}{x^2+y^2} \le \frac{2x^4}{x^2} = 2x^2 \to 0$$