$s = |\alpha_1| + \cdots + |\alpha_n| \neq 0$ $\qquad \beta_k = \dfrac{\alpha_k}{s}$ $\qquad k = 1, \dots, n$ $\qquad \sum_{k=1}^{n} |\beta_k| = \sum_{k=1}^{n} \dfrac{|\alpha_k|}{s} = \cancel{\dfrac{1}{s}} \cancel{\sum_k |\alpha_k|} = 1$

$\| \alpha_1 x_1 + \cdots + \alpha_n x_n \| \geq c\, s$ $\qquad \dfrac{\| \alpha_1 x_1 + \cdots + \alpha_n x_n \|}{s} \geq c$

$\| \frac{\alpha_1}{s} x_1 + \cdots + \frac{\alpha_n}{s} x_n \| \geq c$ $\qquad \| \beta_1 x_1 + \cdots \beta_n x_n \| \geq c$ $\qquad$ Falso.

Si no existe c. $\Rightarrow$ $\quad y_m = \beta_1^{(m)} x_1 + \cdots + \beta_n^{(m)} x_n$ $\quad$ tq $\quad \|y_m\| \xrightarrow{m\to\infty} 0$ $\qquad \|y_m\| < \varepsilon$ ($m \to$ grande)

$\sum_{k=1}^{n} |\beta_k^{(m)}| = 1$ $\qquad \beta_1^{(m_\ell)} \quad \beta_2^{(m_\ell)} \quad \cdots \quad \beta_n^{(m_\ell)}$ $\quad$ son todos acotados !!!

$$\begin{array}{ccc} \beta_1^{(m)} & \beta_2^{(m)} & \\ \downarrow & \downarrow & \\ \beta_1^{(m_{\ell_j})} & \beta_2^{(m_{\ell_j})} & \Rightarrow \exists (\ell_i)_i \text{ tq. } \beta_k^{(\ell_i)} \xrightarrow{i\to\infty} \beta_k \\ \downarrow \ell_j\to\infty & \downarrow j\to\infty & \\ \beta_1 & \beta_2 & \end{array}$$

$(y_m)_m \in M$. 1) $\forall \varepsilon > 0 \quad \exists N \in \mathbb{N} \quad \forall m > N \quad \forall p \in \mathbb{N} \quad \| y_m - y_{m+p} \| < \varepsilon$.

$\| \sum \alpha_k x_k \| \geq c (|\alpha_1| + \dots + |\alpha_n|)$

2) $\exists e_1 \dots e_n \in M$. $\quad \forall m \quad y_m = \alpha_1^{(m)} e_1 + \dots + \alpha_n^{(m)} e_n$

$$\| y_m - y_{m+p} \| = \| (\alpha_1^{(m)} - \alpha_1^{(m+p)}) e_1 + \dots + (\alpha_n^{(m)} - \alpha_n^{(m+p)}) e_n \| \leq \varepsilon$$

$$c \left(\sum_{k=1}^{n} | \alpha_k^{(m)} - \alpha_k^{(m+p)} | \right) \Rightarrow \sum_{k=1}^{n} | \alpha_k^{(m)} - \alpha_k^{(m+p)} | \leq \varepsilon / c$$

$\alpha_k^{(m)} \longrightarrow$ son cauchy !! $\quad \alpha_k^{(m)} \xrightarrow{m} \alpha_k$

$$y = \alpha_1 e_1 + \dots + \alpha_n e_n$$

$$\| y_m - y \| = \| \sum_{k=1}^{n} (\alpha_k^{(m)} - \alpha_k) e_k \| \leq \sum_k \| (\alpha_k^{(m)} - \alpha_k) e_k \| = \sum_k | \alpha_k^{(m)} - \alpha_k | \| e_k \|$$

$$\lim_{m \to \infty} \| y_m - y \| = \lim_{m \to \infty} \sum_{k=1}^{n} \underbrace{| \alpha_k^{(m)} - \alpha_k |}_{\downarrow 0} \| e_k \| = 0$$

$\dim X = n$, $\exists e_1 \cdots e_n$ tq. $\forall x \in X$ $x = \alpha_1 e_1 + \cdots + \alpha_n e_n$

$$\|x\| = \|\sum_{k=1}^{n} \alpha_k e_k\| \geq c \sum_k |\alpha_k| \Rightarrow \sum_{k=1}^{n} |\alpha_k| \leq \frac{\|x\|}{c}$$

$L = \max_{k=1,\dots,n} \|e_k\|'$

$$\|x\|' = \|\sum_k \alpha_k e_k\|' \leq \sum_k \|\alpha_k e_k\|' = \sum_k |\alpha_k| \|e_k\|' \leq L \underbrace{\sum_{k=1}^{n} |\alpha_k|} \leq \underbrace{\frac{L}{c}}_{b} \|x\|$$

$\|x\|' \leq b \|x\|$

Repitiendo ... Deberes! $\|x\| \leq d \|x\|'$

Cerrado + acotado $\Leftrightarrow$ compacto en dimensión finita

M cerrado y acotado. $\dim M = n \Rightarrow \exists e_1 \cdots e_n$ base de M $\Rightarrow \forall (x_m)_m \in M$

$x_m = \alpha_1^{(m)} e_1 + \cdots + \alpha_n^{(m)} e_n$ M acotado $\Rightarrow x_n$ está acotado $\exists K > 0$ $\forall m$

$\|x_m\| \leq K$ $\quad K \geq \|x_m\| = \|\sum_k \alpha_k^{(m)} e_k\| \geq c \sum_k |\alpha_k^{(m)}| \Rightarrow \sum_k |\alpha_k^{(m)}| \leq K/c = K^{\sim}$

$\Rightarrow$ cada una de las suc. $\alpha_k^{(m)}$ es acotada.

$\exists (l_i)_i$ t.q. $\alpha_k^{(l_i)} \xrightarrow{i} \alpha_k \in \mathbb{R}$ $\qquad x = \sum \alpha_k e_k \in M$

$x_{l_i} \to x$ quando $i \to \infty$ $\qquad \| x_{l_i} - x \| \xrightarrow{i \to \infty} 0 \Rightarrow x \in M.$