$\lim_{n\to\infty} x_n = x$ $\qquad x \in X \qquad X = (0,1) \subset \mathbb{R} \qquad x_n = \frac{1}{n} \qquad \lim_{n\to\infty} \frac{1}{n} = 0 \notin X$

Ej. $\lim_{n\to\infty} x_n = x \Rightarrow x$ es único.

Sup $\lim_n x_n = x \qquad \lim_n x_n = y \qquad x \neq y$

$\forall \varepsilon > 0 \ \exists N_1 \in \mathbb{N}$ t.q. $\forall n > N_1$ $\quad \Big| \quad$ $\forall \varepsilon > 0 \ \exists N_2 \in \mathbb{N}$ t.q. $\forall n > N_2$ $\Big\}$ Escoger $N = \max(N_1, N_2)$

$\|x_n - x\| < \varepsilon/2 \qquad \|x_n - y\| < \varepsilon/2$

$\forall n > N \qquad$ si $n > N_2$

$\|x-y\| = \|x - x_n + x_n - y\| \le \|x - x_n\| + \|x_n - y\| < \varepsilon/2 + \varepsilon/2 = \varepsilon \Rightarrow \|x-y\| = 0 \Leftrightarrow x = y$

$$\big|\,\|x\| - \|y\|\,\big| \le \|x-y\| \qquad x_n \to x \qquad \|x - x_n\| \to 0 \qquad \big|\,\|x_n\| - \|x\|\,\big| \to 0$$

$$\Downarrow$$

$$\lim_n \|x_n\| = \|\lim_n x_n\| \quad \Leftarrow \quad \lim_n \|x_n\| \to \|x\|$$

$$\|x\| \le \|x - y + y\| \le \|x-y\| + \|y\| \Rightarrow \|x\| - \|y\| \le \|x-y\|$$

$$-\|x-y\| \le \|x\| - \|y\| \quad \Leftarrow \quad \|y\| - \|x\| \le \|x-y\|$$

$$-\|x-y\| \le \|x\| - \|y\| \le \|x-y\| \Rightarrow \big|\,\|x\| - \|y\|\,\big| \le \|x-y\|$$

$$\mathbb{R}^n \qquad x^{(m)} \to x \qquad x_1^{(m)} \to x_1 \qquad x_2^{(m)} \to x_2 \qquad \dots \qquad x_n^{(m)} \to x_n$$

$$x^{(m)} = \begin{pmatrix} x_1^{(m)} \\ x_2^{(m)} \\ \vdots \\ x_n^{(m)} \end{pmatrix} \longrightarrow \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

$$0 \xleftarrow{m\to\infty} \|x^{(m)} - x\|^2 = \sum_{k=1}^{n} |x_k^{(m)} - x_k|^2 \iff x_k^{(m)} \to x_k \quad \forall k=1,\dots n$$

$$x^{(m)} \in \mathbb{R}^n \qquad \begin{pmatrix} x_1^{(m)} \\ \vdots \\ x_n^{(m)} \end{pmatrix} \qquad x^{(m)} \text{ Cauchy} \Rightarrow \forall \varepsilon > 0 \ \exists N \in \mathbb{N}, \ \begin{matrix}\forall m > N \\ \forall p \in \mathbb{N}\end{matrix} \ \|x^{(m+p)} - x^{(m)}\| < \varepsilon$$

$$\|x^{(m+p)} - x^{(m)}\|^2 = \sum_{k=1}^{n} |x_k^{(m+p)} - x_k^{(m)}|^2 < \varepsilon^2 \Rightarrow \forall k=1,\dots n$$

$|x_k^{(m+p)} - x_k^{(m)}| < \varepsilon \Rightarrow x_k^{(m)} \to$ es de Cauchy. T. Cauchy en $\mathbb{R}$ $\Rightarrow$

$$\lim_{m\to\infty} x_k^{(m)} = x_k \quad \forall k=1,\dots n \qquad \Rightarrow 0 \le \|x^{(m)} - x\|^2 = \underbrace{\sum_{k=1}^{n} (x_k^{(m)} - x_k)^2}_{\downarrow \atop 0} \xrightarrow{m} 0$$

$x = (x_1, \dots, x_n)$

$$\boxed{x^{(m)} \to x}$$