$\rho(x,y) = \|x-y\|$ 1° $\rho(x,y) \geq 0$ $\rho(x,y)=0 \iff x=y$

$\|x-y\|=0 \iff x-y=0$

2° $\rho(x,y) = \|x-y\| = \|(-1)(y-x)\| = |-1|\,\|y-x\| = \rho(y,x)$

3° $\rho(x,z) \leq \rho(x,y) + \rho(y,z)$

$\rho(x,z) = \|x-z\| = \|x \underbrace{-y+y}_{0} - z\| \leq \|x-y\| + \|y-z\| = \rho(x,y) + \rho(y,z)$

$$f(t) = t^{1-p} u^p + (1-t)^{1-p} v^p \qquad t(0,1), \quad u,v \geq 0 \qquad p \geq 1$$

$$\lim_{t \to 0^+} f(t) = +\infty \qquad \lim_{t \to 1^-} f(t) = +\infty$$

$$f'(t) = 0 \Rightarrow f' = \underset{\neq 0}{(1-p)} \left[\underbrace{t^{-p} u^p - (1-t)^{-p} v^p}_{=0} \right] \qquad t^{-p} u^p = (1-t)^{-p} v^p \Rightarrow t^{-1} u = (1-t)^{-1} v$$

$$\Rightarrow \quad t_0 = \frac{u}{u+v}, \quad 1 - t_0 = \frac{v}{u+v}$$

$$f''(t_0) \neq 0 \ (>0) \qquad f'' = (1-p)\left[-p t^{-p-1} u^p + (-p)(1-t)^{-p-1} v^p \right] > 0$$

en t_0 un mínimo!!

$$\inf_{t \in (0,1)} \left(f(t), \ u,v \geq 0, \ p \geq 1 \right) = (u+v)^p$$

$$\|x+y\| \leq \|x\| + \|y\| \qquad \|x\| = \left(\sum_{i=1}^{n} |x_i|^p \right)^{1/p}$$

$$|x_i + y_i|^p \leq \big| |x_i| + |y_i| \big|^p = \inf_{t \in (0,1)} \left(t^{1-p} |x_i|^p + (1-t)^{1-p} |y_i| \right) \leq t^{1-p} |x_i|^p + (1-t)^{1-p} |y_i|^p$$

$$\sum_i |x_i+y_i|^p \le t^{1-p}\sum_i |x_i|^p + (1-t)^{1-p}\sum_i |y_i|^p = t^{1-p}\|x\|^p + (1-t)^{1-p}\|y\|^p$$

$$\inf_{t\in(0,1)} \sum_i |x_i+y_i|^p = \sum_i |x_i+y_i|^p \le \inf_{t\in(0,1)} \left(t^{1-p}\|x\|^p + (1-t)^{1-p}\|y\|^p \right) = (\|x\|+\|y\|)^p$$

$$\sum_i |x_i+y_i|^p = \|x+y\|^p$$

$$\Rightarrow \quad \|x+y\| \le \|x\|+\|y\| \qquad !!!$$