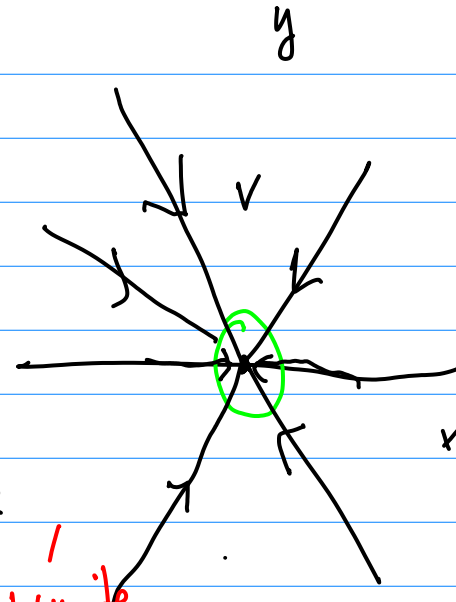
$$f(x,y) = \frac{x^2-y^2}{x^2+y^2} \qquad \lim_{(x,y)\to(0,0)} f(x,y)$$



$y = mx \qquad m \neq 0 \qquad x \to 0 \Rightarrow y \to 0$

$$f(x,mx) = \frac{x^2 - m^2x^2}{x^2+m^2x^2} = \frac{\cancel{x^2}(1-m^2)}{\cancel{x^2}(1+m^2)} = \frac{1-m^2}{1+m^2} \xrightarrow{x\to 0} \frac{1-m^2}{1+m^2}$$

NO TIENE LIMITE

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4+y^2} \qquad y = mx$$

$$f(x,mx) = \frac{x^2 mx}{x^4 + m^2x^2} = \frac{mx}{x^2+m^2}$$

$$m \neq 0 \qquad \lim_{x\to 0} f(x,mx) = 0$$

$$y = x^2 \qquad f(x,x^2) = \frac{x^2 x^2}{x^4+x^4} = \frac{1}{2}$$

$$\lim_{(x,y)\to(0,0)} \frac{|x|^{3/2} y}{x^2+y^2} = ?$$

$$y = mx \qquad \frac{|x|^{3/2} mx}{x^2+m^2x^2} =$$

$$= |x|^{1/2} \underbrace{\frac{m}{1+m^2}}_{\text{acot.}} \underbrace{\frac{x}{|x|}}_{\text{acotado.}} \to 0$$

$$|f| \to 0 \Rightarrow f \to 0$$

$y = \alpha x^2, \quad f(x, \alpha x) = \dfrac{|x|^{3/2}\, \alpha x^2}{x^2 + \alpha^2 x^4} = \dfrac{|x|^{3/2}\, \alpha}{1 + \alpha^2 x^2} \to 0$

$\alpha \neq 0$

$y = x^3 \qquad (0) \qquad 0$

$y = \alpha x^m \qquad m > 0 \qquad (0)$

$0 \le |f(x,y)| \le \alpha(x,y) \to 0$

NO ACOTADO

$\dfrac{|x|^{3/2}\, |y|}{x^2 + y^2} \le \dfrac{|x|^{3/2}\, |y|}{|y|^2} = \dfrac{|x|^{3/2}}{|y|}$

$x \cdot \qquad y = x^{100}$

$y = x^{1/100}$

$\le \dfrac{|x|^{3/2}\, |y|}{|x|^2} = \dfrac{|y|}{|x|^{1/2}}$

$a, b \neq 0$

NO ACOT.

$0 \le (|a| - |b|)^2 = |a|^2 + |b|^2 - 2|ab| \ge 0$

$\dfrac{|ab|}{a^2 + b^2} \le \dfrac{1}{2} \qquad a = |x|^{1/2} \qquad b = |y|$

$|x|^{1/2} \dfrac{|x||y|}{x^2 + y^2} \le \dfrac{1}{2} |x|^{1/2} \longrightarrow 0$