

$$\lim_{(x,y) \rightarrow 0} \frac{x^2 y}{x^2 + y^2} \Rightarrow \left| \frac{x^2 y}{x^2 + y^2} \right| \leq \frac{|xy| \leq \frac{1}{2}}{x^2 + y^2} |x| \leq \frac{1}{2} |x| \rightarrow 0$$

$$dH(f(x,y)) = \frac{\partial f}{\partial x} \Delta x + \frac{\partial f}{\partial y} \Delta y = \underbrace{\begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{pmatrix}}_{Df(x,y)} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{x \rightarrow 0} \frac{f(x,0) - f(0,0)}{x}$$

$$\left| \frac{x^2 y^2}{x^2 + y^2} \right| = \frac{|y|^2}{x^2 + y^2} |x| \leq |y|^2 \rightarrow 0$$

$$\frac{f(x,y) - f(0,0) - Df(0,0) \begin{pmatrix} x \\ y \end{pmatrix}}{\sqrt{x^2+y^2}} = \frac{f(x,y)}{\sqrt{x^2+y^2}}$$

$$\lim_{(x,y) \rightarrow (0,0)} \left(\frac{x^4+y^4}{x^2+y^2} \right) \rightarrow 0$$

$$\frac{x^4+y^4}{x^2+y^2} = \frac{x^4}{x^2+y^2} + \frac{y^4}{x^2+y^2} \leq$$

$$\frac{\lim_{(x,y) \rightarrow (0,0)} \left(\frac{x^4+y^4}{x^2+y^2} \right)}{\sqrt{x^2+y^2}} \sim \frac{x^4+y^4}{(x^2+y^2)\sqrt{x^2+y^2}} \leq \frac{x^2+y^2}{\sqrt{x^2+y^2}} \rightarrow 0$$

$$\leq \frac{1}{\sqrt{x^2+y^2}} = \sqrt{x^2+y^2} \rightarrow 0$$