$$z_{xx} - z_{yy} = 0$$

$x = u + v$, $y = u - v$, $z = w$

I) $\left.\begin{aligned} dx &= du + dv \\ dy &= du - dv \end{aligned}\right\}$ $\quad dx + dy = 2du$, $\quad dx - dy = 2dv$

$du = \dfrac{dx+dy}{2}$ $\quad dv = \dfrac{dx-dy}{2}$

$$dz = dw = w_u du + w_v dv = w_u\left(\frac{dx+dy}{2}\right) + w_v\left(\frac{dx-dy}{2}\right) =$$

$$z_x dx + z_y dy = \underbrace{\frac{1}{2}(w_u + w_v)}_{z_x} dx + \underbrace{\frac{1}{2}(w_u - w_v)}_{z_y} dy \Rightarrow$$

$z_x = \frac{1}{2}(w_u + w_v)$

$z_y = \frac{1}{2}(w_u - w_v)$

II) $$dz_x = \frac{1}{2}\left(w_{uu}du + w_{vu}dv + w_{uv}du + w_{vv}dv\right) = \frac{1}{4}\left(w_{uu}(dx+dy) + w_{uv}(dx-dy) + w_{uv}(dx+dy) + w_{vv}(dx-dy)\right)$$

$$= \underbrace{\frac{1}{4}\left(w_{uu} + w_{uv} + w_{uv} + w_{vv}\right)}_{z_{xx}} dx + \frac{1}{4}\left(w_{uu} - \cancel{w_{uv}} + \cancel{w_{uv}} - w_{vv}\right) = z_{xx}dx + z_{xy}dy$$

$$z_{xx} = \frac{1}{4}\left(w_{uu} + 2w_{uv} + w_{vv}\right)$$

$$dz_y = \frac{1}{2}\left(w_{uu}du + w_{uv}dv - w_{uv}du - w_{vv}dv\right) = z_{xy}dx + z_{yy}dy$$

$$= \frac{1}{4}\Big(w_{uu}(dx+dy) + w_{uv}(dx-dy) - w_{uv}(dx+dy) - w_{vv}(dx-dy) \Big)$$

$$= \frac{1}{4}\big(w_{uu} + w_{uv} - w_{uv} - w_{vv} \big) dx + \underbrace{\frac{1}{4}\big(w_{uu} - w_{uv} - w_{uv} + w_{vv} \big)} dy$$

$$z_{yy} = \frac{1}{4}\big(w_{uu} - 2w_{uv} + w_{vv} \big)$$

III $$z_{xx} - z_{yy} = \frac{1}{4}\big(\cancel{w_{uu}} + 2w_{uv} + \cancel{w_{vv}} \big) - \frac{1}{4}\big(\cancel{w_{uu}} - 2w_{uv} + \cancel{w_{vv}} \big) = w_{uv}$$

La ecuación $z_{xx} - z_{yy} = 0 \Rightarrow w_{uv} = z_{uv} = 0$

$$z_{uv} = 0 \Rightarrow \frac{\partial}{\partial u}(z_v) = 0 \Rightarrow z_v = f(v) \Rightarrow z(v) = \int f(v)\,dv = F(v)$$

$$\frac{\partial}{\partial v}(z_u) = 0 \Rightarrow z_u = g(u) \Rightarrow z(u) = \int g(u)\,du = G(u)$$

$$\frac{\partial^2}{\partial u \partial v}\big(F(v) + G(u) \big) = \cancel{\frac{\partial^2}{\partial u \partial v} F(v)}^{\,0} + \cancel{\frac{\partial^2}{\partial u \partial v} G(u)}^{\,0} = 0 \Rightarrow z = F(v) + G(u)$$

F y G cualesquiera

$$z(x,y) = F(x-y) + G(x+y)$$