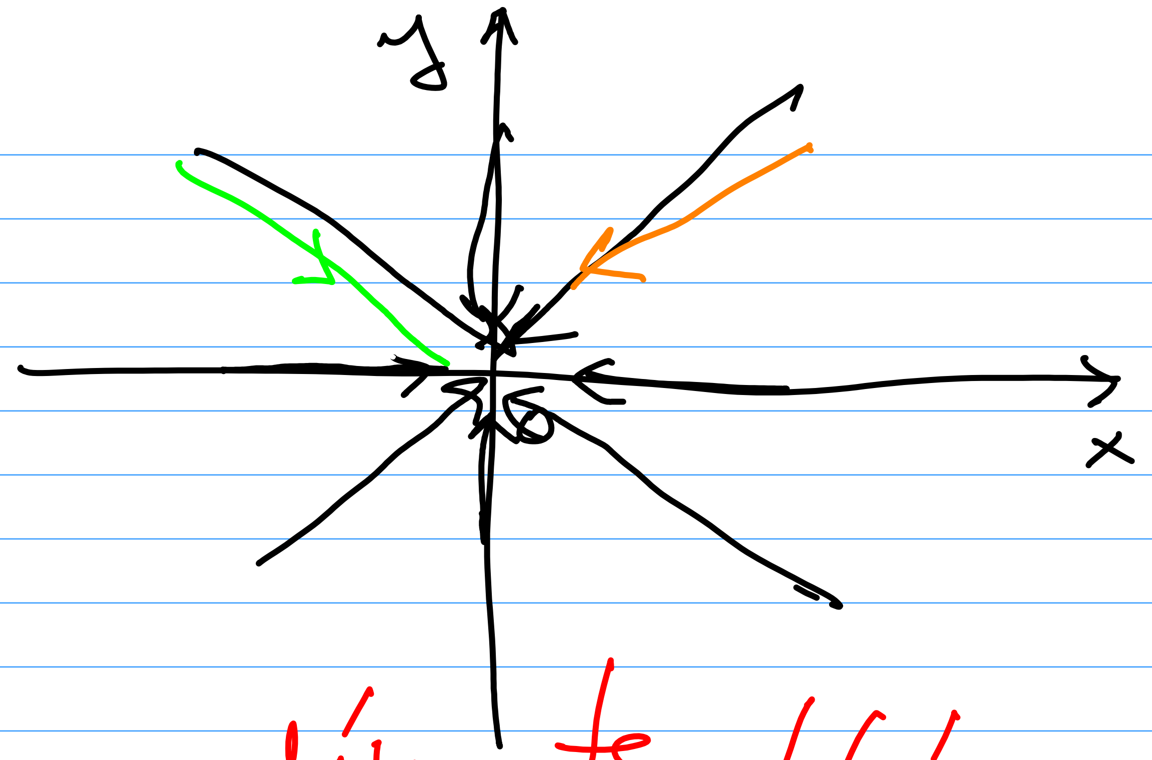
$$f(x,y) = \frac{x^2-y^2}{x^2+y^2} \qquad \lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$$



$y = mx \qquad x \to 0 \qquad y \to 0 \qquad m \neq 0$

$$f(x,mx) = \frac{x^2 - m^2x^2}{x^2+m^2x^2} = \frac{x^2(1-m^2)}{x^2(1+m^2)} = \frac{1-m^2}{1+m^2}$$

NO Tiene límite !!!

$$f(x,y) = \frac{x^2 y}{x^4 + y^2}$$

$\exists \lim_{(x,y)\to 0} f(x,y) = ??$

$$f(x, mx) = \frac{x^2 \cdot mx}{x^4 + m^2 x^2} \qquad m \neq 0$$

$$= \frac{x^3 m}{x^2(x^2 + m^2)} \xrightarrow{x \to 0} \frac{0 \cdot m}{m^2} = 0$$

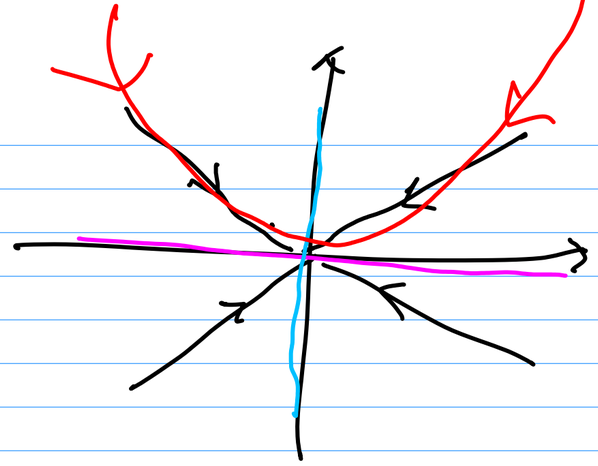
$y = 0$ $\quad x = 0 \rightarrow f(0,y) = 0$

$\downarrow$

$f(x,0) = 0$

$y = x^2$ $\quad f(x, x^2) = \frac{x^2 \cdot x^2}{x^4 + x^4} = \frac{1}{2}$!!

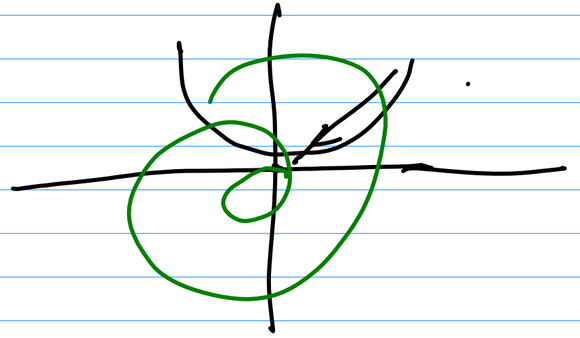
NO TIENE LÍMITE



$$f(x,y) = \frac{|x|^{3/2} y}{x^2+y^2} \qquad \lim_{(x,y)\to 0} f(x,y) = ???$$

$$f(x, y=mx) = \frac{|x|^{3/2} mx}{x^2 + m^2x^2} = |x|^{1/2} \frac{x \cancel{x} m}{\cancel{x^2}(1+m^2)} \to 0$$

$$|x|^{1/2} \frac{|x|\ x}{x\ \cancel{x}} \cdot \frac{m}{1+m^2} \to 0$$



$y = x^2$?

$$\frac{|x|^{3/2} x^2}{x^2 + \cancel{x^4}} \sim |x|^{3/2} \to 0$$

$y = \sqrt{|x|}$

$$\frac{|x|^{3/2} |x|^{1/2}}{\cancel{x^2} + |x|} \sim \frac{|x|^{3/2} |x|^{1/2}}{|x|} = |x| \to 0$$

$|f(x,y) - 0| < \varepsilon$ $\quad \forall$ (x,y) estén cerca del 0

$0 \leq \left| \dfrac{|x|^{3/2}\, y}{x^2+y^2} \right| \leq ? \xrightarrow{?} 0$

$|x| = a \qquad |y| = b$

$(a-b)^2 \geq 0 \qquad 2ab \leq a^2+b^2$

$a^2+b^2-2ab \geq 0 \qquad |ab| \leq \dfrac{a^2+b^2}{2}$

$|x|^{1/2} \dfrac{|x||y|}{x^2+y^2} \leq \dfrac{1}{2}$

$|x||y| \leq \dfrac{1}{2}(|x|^2+|y|^2$

$0 \leq |f(x,y)| \leq \dfrac{|x|^{1/2}}{2} \to 0 \qquad \begin{matrix} x \to 0 \\ y \to 0 \end{matrix}$

$|f(x,y)| \to 0$!!!

$\Downarrow$

$f(x,y) \Rightarrow 0$