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SOME NEW MATRIX FORMULAS RELATED TO HERMITE MATRIX POLYNOMIALS THEORY

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Abstract

In this paper an integral representation for the Hermite matrix polynomials is given. By means of the exact computation of certain matrix integrals and the integral representation of Hermite matrix polynomials, a formula for the generating function of the product of Hermite matrix polynomials is obtained. Both the integral representation of the Hermite matrix polynomials and the formula for the generating function of the product of the Hermite matrix polynomials coincide in the scalar case with the well-known formulas for the classical Hermite polynomials.

1 Introduction

General orthogonal polynomials play an important role in approximation theory, numerical analysis and the analytic solutions of boundary problems for partial differential, see for example [2], [4], [13], [9]. The Hermite polynomials arise in the study of classical boundary value problems in parabolic regions, through the use of parabolic coordinates, or in quantum mechanics, see [9, chapter 6], [20], as well as in other application areas.

Orthogonal polynomials which have matrix coefficients appear in connection with representation theory, matrix expansion problems, prediction theory [1], [3], [11], [18] and in the reconstruction of matrix functions. Those orthogonal on the unit circle are considered in [22], and matrix polynomials orthogonal on the real line have been treated in [21] and [24], and, in the last years in [24], [15], [7], [8]. In [14], [16] and [17], Laguerre, Gegenbauer and Hermite matrix polynomials have been introduced.

If $H_n(x)$ denotes the nth scalar Hermite polynomial, the formulas

$$H_n(x) = \frac{2^n(-i)^n e^{x^2}}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-t^2+2ixt} t^n dt \quad , \quad n = 0, 1, 2, \dots, \tag{1}$$

and

$$W(x, y, t) = (1-t^2)^{-\frac{1}{2}} e^{\frac{\left[2xyt-(x^2+y^2)t^2\right]}{(1-t^2)}} = \sum_{n \geq 0} \frac{H_n(x) H_n(y)}{2^n n!} t^n \quad , \quad |t| < 1 \quad , \tag{2}$$

play a fundamental role in the study of the convergence of the expansions in series of Hermite polynomials, as well as in the computation of the coefficients of such series, see [20, chapter 4].

In the study of matrix functions and series of Hermite matrix polynomials $H_n(x, A)$, it turns out that an integral representation of $H_n(x, A)$, as well as the exact evaluation of the matrix series

$$\sum_{n\geq 0} \frac{H_n(x,A)H_n(y,A)}{2^n n!} t^n \quad ,$$

are necessary. This motivates the search for matrix analogues of formulas (1)-(2), and it constitutes the aim of this paper whose organization is as follows. In section 2 we obtain an integral representation of $\exp\left(-\frac{Ax^2}{2}\right)$ for matrices in $C^{r\times r}$ whose eigenvalues lie in the right-half plane, and this is used to give an integral representation of the Hermite matrix polynomials. In section 3 parametric matrix integrals of the type

$$\int_{-\infty}^{+\infty} \exp(Bxt)\exp\left(-\frac{Ax^2}{2}\right)dx$$

are calculated in a closed form. Finally, in section 4, a matrix formula for the generating function of the product of Hermite matrix polynomials is established.

Throughout this paper the set of all the eigenvalues of a matrix D in $C^{r\times r}$ will be denoted by $\sigma(D)$, and $\|D\|$ will denote the 2-norm of D, defined by

$$\|D\| = \sup_{x\neq 0} \frac{\|Dx\|_2}{\|x\|_2} \quad ,$$

where, for a vector y in C^r, $\|y\|_2$ denotes the usual euclidean norm of y. If D belongs to $C^{r\times r}$ and $\alpha(D) = \max\{Re z \ ; \ z\in\sigma(D)\}$, then by [12, p.556],

$$\|e^{xD}\| \leq e^{x\alpha(D)} \sum_{k=0}^{r-1} \frac{(\|D\|x\sqrt{r})^k}{k!} \quad , \quad x\geq 0 \ . \tag{3}$$

In what follows, the word integrable means integrable in the Lebesgue sense.

2 Integral representation of the Hermite matrix polynomials.

The following result is the starting point of the paper.

Theorem 2.1 *Let A be a matrix in $C^{r\times r}$ satisfying the condition*

$$Re\, z > 0 \quad for \quad every \quad z\in\sigma(A) \ . \tag{4}$$

Then it follows that

$$\exp\left(-\frac{Ax^2}{2}\right) = 2\left(\frac{2A^{-1}}{\pi}\right)^{\frac{1}{2}} \int_0^\infty \exp\left(-2v^2A^{-1}\right)\cos 2vx\,dv \quad , \quad -\infty < x < +\infty \ . \tag{5}$$

Proof. Let us consider the parametric matrix integral

$$J(x) = \int_0^\infty \exp\left(-2A^{-1}v^2\right)\cos 2vx\,dv \quad , \quad -\infty < x < \infty \ , \tag{6}$$

and note that if $z = x+iy$,

$$\frac{1}{z} = \frac{\bar{z}}{|z|^2} \quad and \quad thus \quad Re\left(\frac{1}{z}\right) = \frac{Re(z)}{|z|^2} \ . \tag{7}$$

By the spectral mapping theorem [6, p.569], and (4) and (7), it follows that

$$Re\omega > 0 \quad for \quad every \quad \omega \in \sigma(A^{-1}) \quad . \tag{8}$$

From the inequalities (3) and (8), the absolute convergence of $J(x)$ defined by (6) for any real number x, as well as the uniform convergence of the integral

$$\int_0^\infty v \exp\left(-2A^{-1}v^2\right) \sin 2vx dv \quad ,$$

in every neighbourhood of x. By the Leibnitz theorem for the differentiation of matrix integrals, [5, p.174], [23], and integrating by parts, it follows that

$$J'(x) = -xAJ(x) + \frac{A}{2}\left[\exp\left(-2A^{-1}v^2\right)\sin 2xv\right]_{v=0}^{v=\infty} \quad .$$

Now, again by (3), one gets

$$\left[\exp\left(-2A^{-1}v^2\right)\sin 2xv\right]_{v=0}^{v=\infty} = 0 \quad ,$$

and

$$J'(x) = -xAJ(x) \quad .$$

Taking $x = 0$ in (6), and using the expression (4.8) of [17],

$$J(0) = \int_0^\infty \exp\left(-2A^{-1}v^2\right) dv = \frac{(\pi A)^{\frac{1}{2}}}{2\sqrt{2}} \quad .$$

Thus $J(x)$ is a solution of the initial value problem

$$J'(x) = -xAJ(x) \quad ; \quad J(0) = \frac{1}{2}\left(\frac{\pi A}{2}\right)^{\frac{1}{2}} \quad , \quad -\infty < x < +\infty \quad . \tag{9}$$

Since $D(x) = xA$ satisfies the condition $D(x_1)D(x_2) = D(x_2)D(x_1)$, by [19, p.600], the solution of (9) is

$$J(x) = \exp\left(-\int_0^x sAds\right) J(0) = \exp\left(-\frac{Ax^2}{2}\right)\frac{1}{2}\left(\frac{\pi A}{2}\right)^{\frac{1}{2}} \quad . \tag{10}$$

This proves the equality (5). □

Under the hypothesis (4), by the Leibnitz rule for the differentiation of parametric matrix integrals [5, p.174], and [23], one gets

$$\frac{d^{2n}}{dx^{2n}}J(x) = (-1)^n 2^{2n-1}\int_{-\infty}^{+\infty} \exp\left(-2A^{-1}v^2\right) v^{2n}\, e^{2ixv} dv \quad . \tag{11}$$

Using Rodrigues's formula for the Hermite matrix polynomials [17], it follows that

$$H_{2n}(x,A) = \exp\left(\frac{Ax^2}{2}\right)\left(\frac{A}{2}\right)^{-n}\left[\frac{d^{2n}}{dx^{2n}}\left(\exp\left(-\frac{Ax^2}{2}\right)\right)\right] \quad . \tag{12}$$

By (5) and (12) and the fact

$$\int_{-\infty}^{+\infty} \exp\left(-2A^{-1}v^2\right) v^{2n} \sin 2vxdv = 0 \quad ,$$

it follows that

$$\begin{aligned} H_{2n}(x,A) &= 2\left(\frac{2A^{-1}}{\pi}\right)^{\frac{1}{2}} \exp\left(\frac{Ax^2}{2}\right)\left(\frac{A}{2}\right)^{-n}(-1)^n 2^{2n-1}\int_{-\infty}^{+\infty}\exp\left(-2A^{-1}v^2\right)v^{2n}e^{2ivx}dv \\ &= \frac{1}{\sqrt{\pi}}\exp\left(\frac{Ax^2}{2}\right)\left(\frac{A}{2}\right)^{-(n+\frac{1}{2})}(-1)^n 2^{2n}\int_{-\infty}^{+\infty}\exp\left(-2A^{-1}v^2\right)v^{2n}e^{2ivx}dv \\ &= \frac{\exp\left(\frac{Ax^2}{2}\right)}{\sqrt{\pi}}\left(\frac{A}{2}\right)^{-(n+\frac{1}{2})}(-i)^{2n}2^{2n}\int_{-\infty}^{+\infty}\exp\left(-2v^2A^{-1}+2vixI\right)v^{2n}dv \ . \end{aligned} \quad (13)$$

Similarly, for odd indices we have

$$\frac{d^{2n+1}}{dx^{2n+1}}J(x) = (-1)^{n+1}\frac{2^{2n}}{i}\int_{-\infty}^{+\infty}\exp\left(-2A^{-1}v^2\right)v^{2n+1}\,e^{2ixv}dv \ . \quad (14)$$

By the Rodrigues formula for the Hermite matrix polynomials [17],

$$H_{2n+1}(x,A) = -\exp\left(\frac{Ax^2}{2}\right)\left(\frac{A}{2}\right)^{-(n+\frac{1}{2})}\left[\frac{d^{2n+1}}{dx^{2n+1}}\left(\exp\left(-\frac{Ax^2}{2}\right)\right)\right] \ , \quad (15)$$

so from (5), (14) and (15) and the fact that $(-i)^{2n+1} = i(-1)^{n+1}$,

$$H_{2n+1}(x,A) = \left(\frac{A}{2}\right)^{-(n+1)}\frac{(-2i)^{2n+1}}{\sqrt{\pi}}\exp\left(\frac{Ax^2}{2}\right)\int_{-\infty}^{+\infty}\exp\left(-2v^2A^{-1}+2ixvI\right)v^{2n+1}dv \ . \quad (16)$$

Note that formulas (13) and (16) can be written as a single formula

$$H_n(x,A) = \left(\frac{A}{2}\right)^{-(\frac{n+1}{2})}\frac{2^n(-i)^n\exp\left(\frac{Ax^2}{2}\right)}{\sqrt{\pi}}\int_{-\infty}^{+\infty}\exp\left(-2v^2A^{-1}+2ivxI\right)v^n dv \ . \quad (17)$$

Summarizing, the following integral representation for the Hermite matrix polynomials has been proved:

Theorem 2.2 *Let A be a matrix in $C^{r\times r}$ satisfying condition (4) and let $H_n(x,A)$ be the nth Hermite matrix polynomial. Then the integral representation (17) holds for all $n \geq 0$ and $-\infty < x < +\infty$.*

REMARK 1: For the case $r = 1$, and $A = 2$, expression (17) coincides with formula (1).

3 On the exact computation of certain matrix integrals.

Let us consider the matrix integral

$$\mathcal{Y} = \int_{-\infty}^{+\infty}\exp(Bx)\exp\left(-\frac{Ax^2}{2}\right)dx \ , \quad (18)$$

where A and B are matrices in $C^{r\times r}$ and A satisfies the condition (4). To compute $\mathcal{Y}$, we will use the parametric matrix integral

$$Y(t) = \int_{-\infty}^{+\infty}\exp(Bxt)\exp\left(-\frac{Ax^2}{2}\right)dx \ , \quad -\infty < t < +\infty \ , \quad (19)$$

which, under the condition (4), by (3), is pointwise convergent for any real number t. As well, the matrix integral

$$\mathcal{Y}_0 = \int_{-\infty}^{+\infty} \frac{d}{dt}\left(\exp(Bxt)\exp\left(-\frac{Ax^2}{2}\right)\right) dx \quad ,$$

may be written in the form

$$\mathcal{Y}_0 = \int_{-\infty}^{+\infty} Bx\exp(Bxt)\exp\left(-\frac{Ax^2}{2}\right) dx \quad . \tag{20}$$

Furthermore, from the inequality (3) applied to both $\exp(Bxt)$ and $\exp\left(-\frac{Ax^2}{2}\right)$, the parametric matrix integral (20) is uniformly convergent in every neighbourhood of t lying in the real line. Thus, by the Leibnitz theorem for differentiation of matrix integrals [5, p.174], and taking into account that A commutes with $\exp\left(-\frac{Ax^2}{2}\right)$, it follows that

$$Y'(t) = -B\int_{-\infty}^{+\infty} \exp(Bxt))\left\{-Ax\exp\left(-\frac{Ax^2}{2}\right)\right\} dxA^{-1} \quad . \tag{21}$$

From application of integration by parts in (21), one gets

$$Y'(t) = -B\left\{\left[\exp(Bxt)\exp\left(-\frac{Ax^2}{2}\right)\right]_{x=-\infty}^{+\infty} - Bt\int_{-\infty}^{+\infty} \exp(Bxt)\exp\left(-\frac{Ax^2}{2}\right) dx\right\} A^{-1} \quad . \tag{22}$$

Under the condition (4), from (3) it follows

$$\lim_{x\to\pm\infty} \exp(Bxt)\exp\left(-\frac{Ax^2}{2}\right) = 0 \quad , \quad -\infty < t < +\infty \quad , \tag{23}$$

and by (22) and (23),

$$Y'(t) = B^2tY(t)A^{-1} \quad , \quad -\infty < t < +\infty \quad . \tag{24}$$

Note that taking $t=0$ in (19) and by the expression (4.8) of [17], we have

$$Y(0) = \int_{-\infty}^{+\infty} \exp\left(-\frac{Ax^2}{2}\right) dx = \left(2\pi A^{-1}\right)^{\frac{1}{2}} \quad . \tag{25}$$

From the previous comments it is easy to establish the following result:

Theorem 3.1 *Let A and B be commuting matrices in $C^{r\times r}$ for which A satisfies the condition (4). Then, the matrix integral (19) may be written*

$$Y(t) = \int_{-\infty}^{+\infty} \exp(Bxt)\exp\left(-\frac{Ax^2}{2}\right) dx = \left(2\pi A^{-1}\right)^{\frac{1}{2}} \exp\left(\frac{t^2B^2A^{-1}}{2}\right) \quad .$$

4 On the generating function of products of Hermite matrix polynomials.

In this section we extend formula (2) to the matrix framework, using the integral representation of the Hermite matrix polynomials obtained in section 2. According to (17), for $|t| < 1$ we have

$$\sum_{n\geq 0} \frac{H_n(x,A)H_n(y,A)}{2^n\, n!} t^n$$

$$= \left(\tfrac{A}{2}\right)^{-1} \tfrac{\exp\left(\frac{A}{2}(x^2+y^2)\right)}{\pi} \textstyle\sum_{n\geq 0} \tfrac{\left(-4tA^{-1}\right)^n}{n!}$$
$$\times \quad \textstyle\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \exp\left(-2A^{-1}(u^2+v^2)\right)\exp\left(2iI(ux+vy)\right)(uv)^n dudv \quad . \tag{26}$$

Note that by the inequality (3), for each positive integer n, the matrix function

$$\Phi_n(u,v) = \sum_{h=0}^{n} \frac{\left(-4tuvA^{-1}\right)^h}{h!} \exp\left(-2A^{-1}(u^2+v^2) + 2Ii(ux+vy)\right) \tag{27}$$

is integrable in $\mathcal{R}^2$. If we denote by $\Phi(u,v)$ the non-negative function defined by

$$\Phi(u,v) = \exp\left(4|u||v|\|A^{-1}\| - 2\alpha(A^{-1})(|u|^2+|v|^2)\right)\sum_{k=0}^{r-1} \frac{\left[2\sqrt{r}(|u|^2+|v|^2)\|A^{-1}\|\right]^k}{k!} \quad , \tag{28}$$

for $(u,v) \in \mathcal{R}^2$, then by (3), (27) and (28), it follows that

$$\|\Phi_n(u,v)\| \leq \Phi(u,v) \quad , \quad (u,v) \in \mathcal{R}^2 \quad , \quad n \geq 0 \quad .$$

From (8), it is clear that $\alpha(A^{-1}) > 0$ and $\Phi(u,v)$ is integrable in $\mathcal{R}^2$. By the dominated convergence theorem [9, p.83], we can change the order of summation and integration in (26), obtaining

$$\sum_{n\geq 0} \frac{H_n(x,A)H_n(y,A)}{2^n\, n!} t^n = \left(\frac{A}{2}\right)^{-1} \frac{\exp\left(\frac{A}{2}(x^2+y^2)\right)}{\pi}$$
$$\times \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \exp\left(-2A^{-1}(u^2+v^2)\right)\exp\left(2iI(ux+vy)\right)\exp\left(-4tuvA^{-1}\right) dudv \quad . \tag{29}$$

Using the Fubini-Tonelli theorem [10, p.65] we can write

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \exp\left(-2A^{-1}(u^2+v^2)\right)\exp\left(2iI(ux+vy)\right)\exp\left(-4tuvA^{-1}\right) dudv$$

$$= \int_{-\infty}^{+\infty} \exp\left(-2A^{-1}v^2 + 2ivyI\right)\left\{\int_{-\infty}^{+\infty} \exp\left(-2A^{-1}u^2 + 2u\left(ixI - 2tA^{-1}v\right)\right) du\right\} dv \quad . \tag{30}$$

Let us consider the matrix integral

$$L = \int_{-\infty}^{+\infty} \exp\left(-2A^{-1}u^2 + 2u\left(ixI - 2tA^{-1}v\right)\right) du \quad ; \tag{31}$$

after the change of variable $\omega = 2u$, this can be written in the form

$$L = L(1) = \frac{1}{2}\int_{-\infty}^{+\infty} \exp\left(B_0\omega\right)\exp\left(-\frac{\omega^2}{2}A^{-1}\right)d\omega \quad , \tag{32}$$

where

$$B_0 = ixI - 2tA^{-1}v \tag{33}$$

Since B_0 commutes with A^{-1}, and under the hypothesis (4) its eigenvalues satisfy (8). By (32) and (33), where A^{-1} plays the role of A, and t is replaced by 1, it follows that

$$L = \left(\frac{\pi A}{2}\right)^{\frac{1}{2}}\exp\left(-\frac{Ax^2}{2}\right)\exp\left(2t^2v^2A^{-1} - 2txviI\right) \quad . \tag{34}$$

By (34) and (30),

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \exp\left(-2A^{-1}(u^2+v^2)\right)\exp\left(2ix(u+v)I\right)\exp\left(-4tuvA^{-1}\right)dudv$$

$$= \left(\frac{\pi A}{2}\right)^{\frac{1}{2}}\exp\left(\frac{-x^2A}{2}\right)\int_{-\infty}^{+\infty}\exp\left(-2v^2A^{-1}(1-t^2) + 2vi(y-tx)\right)dv \quad . \tag{35}$$

Let us consider now the matrix integral

$$S = \int_{-\infty}^{+\infty}\exp\left(-2v^2A^{-1}(1-t^2) + 2vi(y-xt)I\right)dv \quad . \tag{36}$$

Under the change of variable $\omega = 2v\sqrt{1-t^2}$ in (36),

$$S = \frac{1}{2\sqrt{1-t^2}}\int_{-\infty}^{+\infty}\exp\left(B_1\omega(y-xt)\right)\exp\left(-\frac{\omega^2A^{-1}}{2}\right)d\omega \quad , \tag{37}$$

where

$$B_1 = \frac{iI}{\sqrt{1-t^2}} \quad . \tag{38}$$

By (37) and (38), where A^{-1} plays the role of A, and $y - xt$ replaces t, it follows that

$$S = \frac{(2\pi A)^{\frac{1}{2}}}{2\sqrt{1-t^2}}\exp\left(\frac{(y-xt)^2B_1^2A}{2}\right) = \left(\frac{\pi A}{2(1-t^2)}\right)^{\frac{1}{2}}\exp\left(\frac{-A(y-xt)^2}{2(1-t^2)}\right) \quad . \tag{39}$$

Consequently, by (35), (37) and (39),

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\exp\left(-2A^{-1}(u^2+v^2)\right)\exp\left(2iI(ux+vy)\right)\exp\left(-4tuvA^{-1}\right)dudv$$

$$= \frac{\pi A}{2\sqrt{1-t^2}}\exp\left(-\frac{Ax^2}{2}\right)\exp\left(-\frac{A(y^2+x^2t^2-2yxt)}{2(1-t^2)}\right) \quad . \tag{40}$$

Hence, from (29) and (40)

$$\sum_{n\geq 0}\frac{H_n(x,A)H_n(y,A)}{2^n\, n!}t^n = (1-t^2)^{-\frac{1}{2}}\exp\left(A\frac{\left[xyt - \frac{1}{2}(x^2+y^2)t^2\right]}{(1-t^2)}\right) \quad .$$

Thus, the following result has been established:

Theorem 4.1 *Let A be a matrix in $C^{r\times r}$ satisfying the condition (4), and let $H_n(x,A)$ be the nth Hermite matrix polynomial. Then, for any real value of t, with $|t|<1$, and for any real values of x and y,*

$$\begin{aligned} W(x,y,t,A) &= (1-t^2)^{-\frac{1}{2}} \exp\left(\frac{A}{2}\frac{\left[2xyt-(x^2+y^2)t^2\right]}{(1-t^2)}\right) \\ &= \sum_{n\geq 0} \frac{H_n(x,A)H_n(y,A)}{2^n\, n!} t^n \quad , \quad |t|<1 \quad . \end{aligned} \tag{41}$$

REMARK 2: For the case $r=1$, and taking $A=2$, the formula (41) coincides with (2).

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